Magnetic hysteresis modeling from measured data using fuzzy logic

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Abstract: This paper propose an accurate Fuzzy model for describing dynamic hysteresis of ferromagnetic material from measured data using soft computing approaches. We highlighted a fuzzy dynamic model based on measured and normalized input/output data on a C core transformer made of 0.33mm laminations of cold rolled SiFe. Membership's functions of fuzzy rules are obtained by using the Expectation-Maximization algorithm and the number of fuzzy rules is optimized by the partition coefficients (PC) and entropy classification (EC) indices. Results show the efficiency of clustering techniques in the identification of highly nonlinear systems

Keywords**:** Magnetic hysteresis, fuzzy clustering, Expectation-Maximization algorithm, Model identification, Dynamic behavior

1. INTRODUCTION

Magnetic materials are fundamental in the construction of electrical machinery, where the accurate determination and analysis of the magnetic field should take hysteresis into account.

Hysteresis phenomenon refers to delay between magnetic field application and magnetization apparition and causes electric losses in used material. From mathematical and physics viewpoint, the dynamic magnetic hysteresis characteristic is a extremely nonlinear and multi-valued relationship between the magnetic field intensity *H(t)* and the magnetization of the magnetic materials *M(t)*. Taking hysteresis into account by developing qualitative model allow a precise determination of various parameters involved in electrical machines analysis.

Many models of magnetic hysteresis have been proposed. Generally, there are two main approaches, phenomenological (Jiles and al, 1986) based on knowledge of physical phenomena occurring in the material to extract behaviour general rules and empirical formulation (Preisach,F 1935) in which we do not care about the physics of phenomena but a mathematical expression. In both case, the determination of model parameters is a relatively difficult task for which specific methods are usually developed.

Artificial intelligence has been applied to modeling hysteresis and identification of model parameters. Wilson *et al* (Wilson. K. P. R and al 2001), uses genetic algorithm for optimizing Jiles-Atherton hysteresis model, Salvini *et al* have developed a neuro-genetic approach for modeling dynamic hysteresis in the harmonic regime (Salvini,A and al 2003), Francesco Riganti *et al* (Francisco,R and al 2005) have proposed an identification method of Jiles-Atherton hysteresis model parameters based on a partnership of heuristic techniques (Genetic Algorithm) and fuzzy logic. Mordjaoui *et al* (Mordjaoui, M and al 2007), (Mordjaoui, M and al 2008), have used an adaptive neuro-fuzzy inference system that does not require a supervised learning for modeling hysteresis function versus frequency. Our contribution consists in developing a qualitative model of dynamic hysteresis of ferromagnetic materials based on Expectation-Maximization clustering algorithm. The performance of the proposed model was investigated using experimental dynamic data.

2. MODELING PROBLEM

Model identification from measured data has become an important tool in several fields of engineering and science because of the complexity of some problems encountered in reality. The traditional approaches of modeling are proved to be insufficient and not very efficient on the accuracy and the exact description of the system behaviour. However, the use of mathematical tools such as the differential or integral equations, algebraic equations, and transfer functions and so on is suitable and justified for defined systems. When the complexity increases, these tools become insufficient whereas the fuzzy approach might be a very useful alternative. While the introduction of fuzzy sets, some researchers have been applying this theory to system identification and function approximation. Several techniques have been developed using artificial intelligence as neural network, clustering approach and other methods. Therefore, fuzzy identification has become a central key in fuzzy system theory. The modeling framework considered in this study is based on the clustering techniques which describe relationship between variables of studied system by means of Takagi Sugeno fuzzy ‘if-then’ rules. In the following, we introduce the c-means algorithm and the Expectation-Maximization approach that we used to hysteresis modeling.

3 FUZZY CLUSTERING ANALYSIS

The purpose of cluster analysis is the partition of data into groups based on common similarities, from a large data set to produce a concise representation of a system’s behavior. It is the most important unsupervised learning methods; they don’t use prior class identifier and do not rely on assumptions common to statistical classification methods. It can be used in modeling and in identification of nonlinear systems. A group can be defined as a collection of objects which are comparable between them and different to the objects belonging to other groups. Our attention here will be focused on clustering of quantitative data which consist on physical process observations. Clustering algorithms may use an objective function to measure the desirability of partitions. Nonlinear optimization algorithms are used to search for local optima of the objective function. A Fuzzy partition of data set *Z* can be represented by a *cxN* matrix $U=[\mu_{ik}]$ called fuzzy partition matrix, where μ_{ik} denotes the degree of membership that the kth observation belongs to the cth cluster and the i^{th} row of *U* contains values of membership function of the i^{th} fuzzy groups of *Z.*

3.1 C-means approach

Several Fuzzy clustering approaches have been used for identification. The Fuzzy c-means is widely used, it based on minimization of cost function given by:

$$J_{FCM}(Z;U,V)= \sum_{i=1}^{c}\sum_{k=1}^{N}(\mu_{ik})^m D_{ikA}^2 \tag{1}$$

$$\mu_{ik}\in[0,1] \quad , \quad 0<\sum_{k=1}^{N}\mu_{ik}<N, \quad \sum_{i=1}^{c}\mu_{ik}=1 \tag{2}$$

$$V=[v_1,v_2,...,v_c]\in R^n$$

Where *V* is a matrix of cluster centers, which have to be determined, and the squared inner-product distance norm is done by the following:

$$D_{ikA}=\|z_k-v_i\|_A^2=(z_k-v_i)^T A(z_k-v_i) \tag{3}$$

$$1\le i\le c, \quad 1\le k\le N$$

It is defined by matrix *A* and $m\in[1,\infty]$ is a weighting exponent which determines the fuzziness of the resulting clusters.

The distance norm in the c-means algorithm is not adaptive and it is often Euclidean. However, it can find only clusters with the same shape and size.

3.2 The Expectation-Maximization (EM) clustering approach

The EM algorithm proposed by Abonyi is a modified Gath-Geva approach (Abonyi, J and al 2007), it is a supervised clustering technique. With this approach, we can circumvent the problem of non-zero off covariance matrix diagonal elements. However, each cluster is described by an input distribution, a local model and an output distribution.

The procedure of Expectation-Maximization algorithm can be summarised as follow:

Given the data set $\mathbf{Z=\{z_{k1}, z_{k2},\ldots, z_{kn}\}^T}$

Choose the number of clusters 1<c<N

Select the weighting exponent (m=2) and the stop criterion (ε >0).

Initialize the partition matrix such that:

$$\mu_{ik}\in[0,1] \quad ; \quad \forall i,k; \quad \sum_{i=1}^{c}\mu_{ik}=1; \; 0<\sum_{k=1}^{N}\mu_{ik}<N, \quad \forall i$$

Repeat for the iteration number l=1,2,…

Step1.

Calculate the clusters parameters:

1: Compute the clusters centers.

$$\mathbf{v}_i^x=\frac{\sum_{k=1}^{N}\mu_{i,k}^{(l-1)}\mathbf{x}_k}{\sum_{k=1}^{N}\mu_{i,k}^{(l-1)}}$$

2: the variance of the Gaussian membership functions:

$$\sigma_{i,j}^2=\frac{\sum_{k=1}^{N}\mu_{i,k}^{(l-1)}\left(x_{j,k}-v_{j,k}\right)^2}{\sum_{k=1}^{N}\mu_{i,k}^{(l-1)}}$$

3: compute parameters of the local models:

$$\theta_i=\left(\mathrm{X}_e^T U_i \mathrm{X}_e\right)^{-1}\mathrm{X}_e^T\,\mathrm{U}_i y$$

Where U_i is the weights matrix having the membership degrees in his main diagonal defined by:

$$U_i=\begin{bmatrix}\mu_{i,1} & 0 & \cdots & 0\\ 0 & & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \mu_{i,N}\end{bmatrix}, \; X=\begin{bmatrix}X_1^T\\ \cdot\\ \cdot\\ X_N^T\end{bmatrix}, y=\begin{bmatrix}y_1\\ \cdot\\ \cdot\\ y_N\end{bmatrix}$$

and the extended matrix X_e is given by:

$$X_e=[X \quad 1]$$

4: a priori of probabilities of the cluster

$$\alpha_i = \frac{1}{N}\sum_{k=1}^{N}\mu_{i,k}$$

5: Weights of the rules

$$w_i = \prod_{j=1}^{n}\frac{\alpha_i}{\sqrt{2\pi\sigma_{i,j}^2}}$$

Step2:

Compute the distance measure.

$$\frac{1}{D_{i,k}^2} = w_i.\exp\left(-\frac{1}{2}\frac{\left(x_{j,k}-v_{i,j}\right)^2}{\sigma_{i,j}^2}\right).$$

$$\frac{1}{\sqrt{2\pi\sigma_i^2}}.\exp\left(-\frac{\left(y_k - f_i(\mathrm{x}_k,\theta_i)\right)^T\left(y_k - f_i(\mathrm{x}_k,\theta_i)\right)}{2\sigma_i^2}\right)$$

where $f_i(\mathrm{x}_k,\theta_i)$ are the local consequent models.

Step3:

Update the partition matrix:

$$\mu_{ik} = \frac{1}{\sum_{j=1}^{c}(D_{ikA}^2 / D_{jkA}^2)^{1/(m-1)}}, \qquad 1 \le i \le c, \quad 1 \le k \le N,$$

Until $\|U^{(l)} - U^{(l-1)}\| \le \varepsilon$

End

4. HYSTERESIS MODELING PROCESS

In order to illustrate the fuzzy modeling procedure of dynamic magnetic hysteresis, we used for training and testing four measured dynamic cycles (4x101 pairs) on a C core transformer made of 0.33mm laminations of cold rolled SiFe at 50 Hz comes from a research report (Saghafifar,M and al 2002) .The biggest loop is the normalised major hysteresis loop and the rest are minor loops (Fig.1) which seen as a highly non-linear relationship between a normalized magnetic induction according to the applied magnetic field *B=f(H)*.

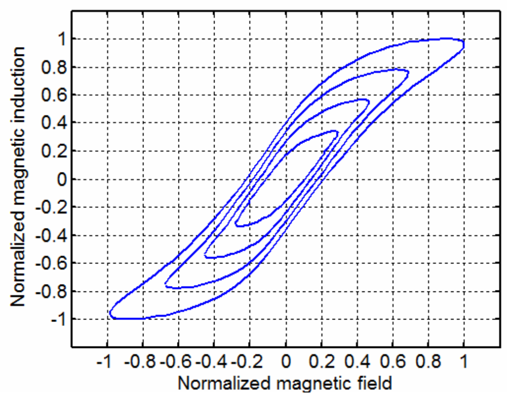
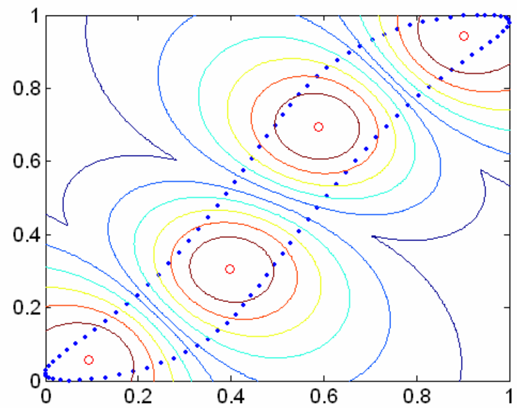
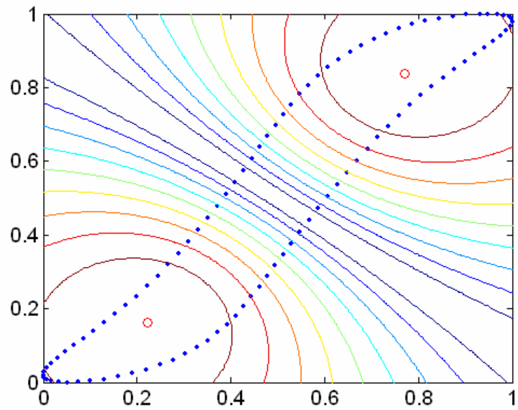


Fig. 1. Measured hysteresis loops

The hysteresis identification has been performed for different regions which correspond to the number of cluster. The results of data hysteresis clustering by using Expectation maximization algorithm for c=4 and c=2 are shown respectively in figures 2 a and b.



-a-



-b-

Fig. 2. Results of EM algorithm by the major hysteresis cycle data set for c=4 (a) and c=2 (b)

Figure.3 shows the corresponding major hysteresis cycle obtained by training for c=4.

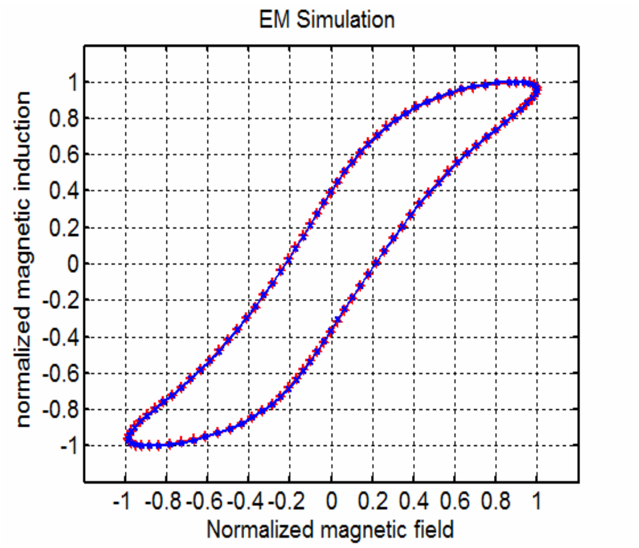


Fig. 3. Measured and simulated hysteresis cycle for c=4

4.1 Approximation and evaluation

To evaluate the quality of approximation (Numerical performance) obtained by the Fuzzy models, we use the following criteria:

4.1.1 Root mean square error (RMSE)

It is an aggregate measure on the total number of items on the deviation from the expected value. Its best value is zero; it is defined by the following expression:

$$RMSE = \sqrt{\frac{1}{N}\sum_{k=1}^{N}(y_k - \hat{y}_k)^2} \qquad (4)$$

where $1 \leq k \leq N$ is the number of points considered for modeling, y is the desired normalized magnetic induction and $\hat{y}$ is the output of the model.

4.1.2 Variance-Accounting for (VAF)

Introduced by Babuska *et al*. (Babuska. R and al 1998), this measure allows the evaluation of model quality; it's a percentage of a model in measuring the standard deviation of the variance between two signals. Its best value is 100% when the two signals are equal, if different, the VAF is lower. The criterion VAF is given by:

$$VAF = 100\% \left[1 - \frac{\operatorname{var}(y - \hat{y})}{\operatorname{var}(y)}\right] \qquad (5)$$

4.1.3 Time calculation Tcal

It is the time machine required for fuzzy model calculations and construction from data.

Table.1 summarizes the quality of model identification (VAF, RMSE and computing time) from various stages of identifications using Matlab toolbox proposed in (Babuska. R and al 2002) (Balasko. B and al) in a micro-computer Pentium 4, 2.06 GHz with 256 MB of RAM and an operating system Windows 2000 Professional. The best model with respect to the RMSE and time computing criterion is for four clusters with four rules consequents (in red, table.1).

Table1: Results of clustering performance of dynamic magnetic hysteresis by Expectation-maximization algorithm

Parameters	*c*=2	*c*=3	*c*=4
VAF	99.7023	99.9982	99.999
RMSE	0.0403	0.0031	0.0023
Tcal (s)	0.4530	0.359	0.4690

The Fuzzy rules consequents which illustrate the behaviour of the optimal local models are illustrated in Table 2 and the corresponding clusters centers in table 3.

Table.2: fuzzy model rules obtained with EM algorithm for four clusters

If $y(k-1)$ is A_{11} and $u(k)$ is A_{12}	$y(k) = 8.44.10^{-1}.y(k-1)+ 8.20.10^{-2}.u(k)-8.39.10^{-2}$
If $y(k-1)$ is A_{21} And $u(k)$ is A_{22}	$y(k) = 8.56.10^{-1}.y(k-1)+ 2.98.10^{-1}.u(k)-6.65.10^{-4}$
If $y(k-1)$ is A_{31} and $u(k)$ is A_{32}	$y(k) = 8.52.10^{-1}.y(k-1)+ 1.47.10^{-1}\,u(k) +3.95.10^{-2}$
If $y(k-1)$ is A_{41} and $u(k)$ is A_{42}	$y(k) = 8.41.10^{-1}.y(k-1)+ 8.04.10^{-2}\,u(k) +8.73.10^{-2}$

Table.3: Clusters centers of EM hysteresis model

$v_{y(k-1)}$	$v_{u(k)}$
$-8.03.10^{-1}$	$-7.41.10^{-1}$
$-7.55.10^{-2}$	$-5.45.10^{-2}$
$5.89.10^{-1}$	$4.35.10^{-1}$
$8.75.10^{-1}$	$8.47.10^{-1}$

In order to optimise the number of fuzzy rules, cluster validity indices are used. This can be accomplished by validation analysis using fuzzy cluster validity indices which are:

4.2 Cluster validity of dynamic magnetic hysteresis fuzzy model

In each partitioning problem the number of clusters must be given by the user prior to the calculation, but it is not often known a priori, in this case it must be determined using validity indices (Abonyi, J and al 2007). Different scalar validity measures have been proposed in the literature, none of them is perfect by itself thus it is suitable to use some indices simultaneously.

4.2.1 Partition coefficient (PC:

Measures the ammount of overlapping between clusters. It use only the membership values of the fuzzy partition of data defined by Bezdek (Bezdek, J.C and al 1975) as follows:

$$PC(c) = \frac{1}{N}\sum_{i=1}^{c}\sum_{k=1}^{N}(\mu_{ik})^2 \qquad (6)$$

where μ_{ik} is the membership of data point *i* in cluster *k*. The main drawback of this indice is the lack of direct connection to the data itself. The optimal number of clusters can be found by the maximum value.

4.2.2 Classification Entropy (CE):

measures only the fuzzyness of the cluster partition, which is similar to the Partition Coefficient.

$$CE(c) = -\frac{1}{N}\sum_{i=1}^{c}\sum_{k=1}^{N}\mu_{ik}\log(\mu_{ik}) \qquad (7)$$

4.2.3 Dunn's Index (DI):

This index was originally designed for the identification of hard partitioning clustering. Therefore, the result of the clustering has to be recalculated.

$$DI(c) = \min_{i\in c}\left\{\min_{j\in c, i\neq j}\left\{\frac{\min_{x\in C_i, y\in C_j} d(x,y)}{\max_{k\in c}\left\{\max_{x,y\in C} d(x,y)\right\}}\right\}\right\} \qquad (8)$$

The main disadvantage of the Dunn’s index is the very expansive computational complexity as c and N increase.

4.2.4 Alternative Dunn index (ADI):

To simplify the calculation of the Dunn index, the Alternative Dunn Index was designed.

$$ADI(c) = \min_{i\in c}\left\{\min_{j\in c, i\neq j}\left\{\frac{\min_{x\in C_i, y\in C_j}\left|d(y,v_i)-d(x_i,v_j)\right|}{\max_{k\in c}\left\{\max_{x,y\in C} d(x,y)\right\}}\right\}\right\} \qquad (9)$$

4.2.5 Partition Index (SC): express the ratio of the sum of compactness and separation of the clusters. Each cluster is measured with the cluster validation method. It is normalized by dividing it by the fuzzy cardinality of the cluster. To receive the partition index, the sum of the value for each individual cluster is used.

$$SC(c) = \sum_{i=1}^{c}\frac{\sum_{k=1}^{N}(\mu_{ik})^m\left\|X_k - v_i\right\|^2}{\sum_i\sum_{j=1}^{c}\left\|v_j - v_i\right\|^2} \qquad (10)$$

SC is useful when comparing different partitions with the same number of clusters. A lower value of SC means a better partition.

4.2.6 Separation Index (S):

In contrast with the partition index (SC), the separation index uses a minimum-distance separation for partition validity.

$$S(c) = \frac{\sum_{i=1}^{c}\sum_{k=1}^{N}(\mu_{ik})^2\left\|x_k - v_i\right\|^2}{N.\min_{i,j}\left\|v_j - v_i\right\|^2} \qquad (11)$$

4.2.7 Xie and Beni’s Index (XB):

Express the quotient of the total variation inside the clusters and the separation of the clusters. The optimal number of clusters should minimize the value of the index.

$$XB(c) = \frac{\sum_{i=1}^{c}\sum_{k=1}^{N}(\mu_{ik})^m\left\|x_k - v_i\right\|^2}{N.\min_{i,k}\left\|x_k - v_i\right\|^2} \qquad (12)$$

The major magnetic hysteresis cycle data is used to find out the optimal number of clusters. During the optimization parameters were fixed to the following values: weighting component m = 2, c ∈ [2… 20]. The values of the validity measures (PC, CE, DI ADI, SC, S and XB) depending from the number of cluster are depicted in (fig.4, fig.5, fig.6) which shows an apparent relationship between the accuracy and the complexity of the models. The main drawback of PC shows that c=2 can be considered as the optimum number of clusters which is confirmed by PC, CE in figure 4, DI in figure 5 and partition index SC too in Figures.6.

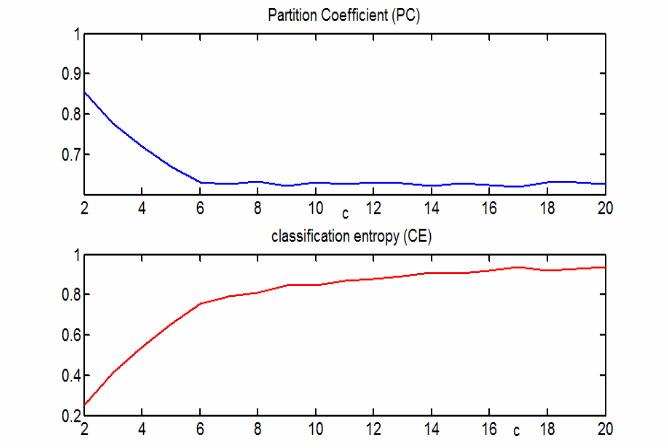


Fig. 4. Value of PC and CE coefficient

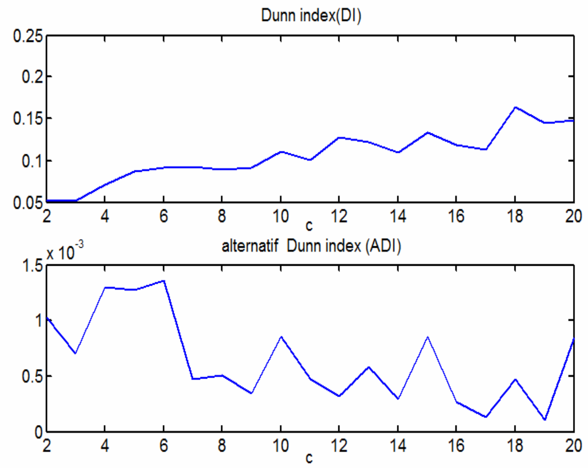


Fig. 5. Value of DI and ADI coefficient

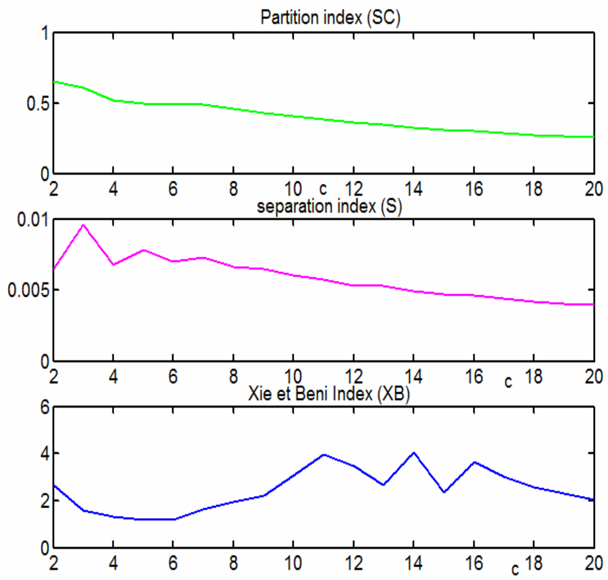


Fig.6: Value of SC, S and XB indices

Expectation-Maximization algorithm is used to construct fuzzy model with two clusters. However, four cycles are considered, the major hysteresis cycle is used for training step and the other minor cycles for testing step. Results of the training and checking error of the obtained fuzzy model are shown in table 4. The corresponding fuzzy membership's functions are illustrated in figure 7. Figure 8 illustrates the comparison of fuzzy model output and measured outputs.

Table.4: Training and checking error

	RMSE	*VAF*
Cycle 1	*0.0205*	*99.9499*
Cycle *2*	*0.0074*	*99.9832*
Cycle 3	*0.0119*	*99.9239*
Cycle 4	*0.0114*	*99.7816*

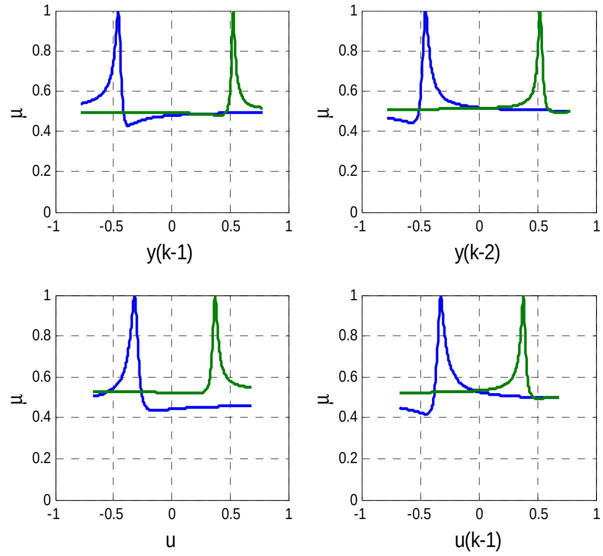


Fig. 7. Membership's functions obtained by the projection of clusters by EM algorithm

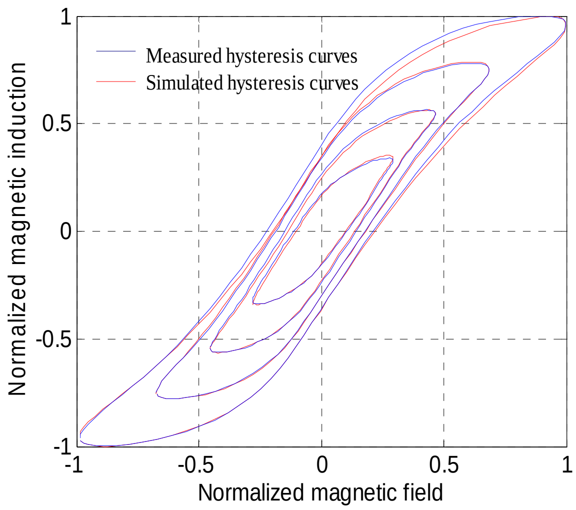


Fig. 8: Dynamic Hysteresis cycle measured and Simulated with EM algorithm

5. CONCLUSIONS

An original dynamic magnetic hysteresis model has been presented using a fuzzy clustering approach. Numerical comparison with experimental results assess the validity of the proposed model and it has shown a good numerical accuracy. Furthermore, the presented model demonstrate that the Expectation-Maximization approach can provide an efficient computational model and reduction of complexity with respect to other classical model. Thus, fuzzy clustering technique is useful for the development of a magnetic material hysteresis model suitable for electrical machines analysis, power electronics and electronic circuits and it is suitable for computer-aided design of .electromagnetic devices.

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