Quantum Mechanical Treatment of the Lamb Shift without Taken into Account the Electric Charge.

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Abstract: We present the calculus of the Lamb shift by using an equivalent expression for the Coulomb interaction energy, on the form $\alpha\hbar c/R$, where α is the fine structure constant. This expression was found by using a new Hamiltonian of interaction between fermions. The obtained results are in a good agreement with experimental data. The calculus was fulfilled in both three dimensional and two dimensional spaces.

Keywords: electron-electron interaction via bosons; Lamb shift; two dimensional space.

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1. INTRODUCTION

The Lamb shift (Lamb and Retherford, 1947) is a small difference in energy between two energy levels $^2S_{1/2}$ and $^2P_{1/2}$ of the hydrogen atom by an amount now known to be *E/h* = 1057.864 MHz. This result is in contradiction with the Dirac and Schrödinger theory which shown that the states with the same *n* and *j* quantum numbers but different *l* quantum numbers ought to be degenerate. The effect is explained by the theory of quantum electrodynamics (Bethe, 1947; Welton, 1948; Greiner and Reinhardt, 1994), in which the electromagnetic interaction itself is quantized. It is assumed that the ground state of the electromagnetic field is not zero, but rather the field undergoes "vacuum fluctuations" that interact with the electron. The contributions to this effect come from the vacuum polarization, electron mass renormalization and anomalous magnetic moment. Often the "vacuum" is a "refuge" for speculations in science.

By using a Hamiltonian of interaction between fermions based on the coupling through flux lines (Dolocan et al, 2005) we have found an equivalent expression for the Coulomb energy of interaction (Dolocan et al,2010), on the form $\alpha\hbar c/R$, where α is the fine structure constant.
In the interaction picture the effective Hamiltonian is given by (Dolocan et al, 2005; 2010)

$$H_I^{eff} = H_{I1}^{eff} + H_{I2}^{eff}$$

$$H_{I1}^{eff} = \hbar \sum_{\mathbf{q},\mathbf{q_o},\mathbf{k}} \left|\mathbf{g}_{\mathbf{q_o}}\right|^2 \frac{\omega_{\mathbf{q}}}{\left(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}-\mathbf{q}}\right)^2 - \omega_{\mathbf{q}}^2} \times$$

$$\left(a_{\mathbf{q}} a^+_{\mathbf{q_o}} a_{\mathbf{q}'_o} a^+_{\mathbf{q}'} + a^+_{\mathbf{q}} a_{\mathbf{q_o}} a^+_{\mathbf{q}'_o} a_{\mathbf{q}'}\right) c^+_{\mathbf{k}-\mathbf{q},\sigma} c^+_{\mathbf{k}',\sigma'} c_{\mathbf{k},\sigma} c_{\mathbf{k}'-\mathbf{q},\sigma'}$$

$$H_{I2}^{eff} = 2\hbar \sum_{\mathbf{q},\mathbf{k}} 2\left|\mathbf{g}_{\mathbf{q}}\right|^2 \frac{1}{\left(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}-\mathbf{q}}\right) - \omega_q} a^+_{\mathbf{q}} a_{\mathbf{q}} c^+_{\mathbf{k}-\mathbf{q},\sigma} c_{\mathbf{k}-\mathbf{q},\sigma} \tag{1}$$

The expectation value of the energy of H_{I1}^{eff} (1) is

$$E_{\text{int}} = \hbar \sum_{\mathbf{q},\mathbf{q_o},\mathbf{k}} 2\left|g_{\mathbf{q_o}}\right|^2 \frac{\omega_{\mathbf{q}}}{\left(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}-\mathbf{q}}\right)^2 - \omega_{\mathbf{q}}^2} \left(n_{\mathbf{q}} + 1\right)\left(n_{\mathbf{q_o}} + 1\right) \times$$

$$n_{\mathbf{k}-\mathbf{q},\sigma} n_{\mathbf{k},\sigma'} \tag{2}$$

and the expectation value of the energy of the Hamiltonian H_{I2} (1) is

$$E_{I2} = 4\hbar \sum_{\mathbf{q},\mathbf{q_o},\nu'} \left|g_{\mathbf{q_o}}\right|^2 \frac{1}{\left(\varepsilon_{\nu'} - \varepsilon_{\nu}\right) - \omega_{\mathbf{q}}} (n_{\mathbf{q}} + 1) n_{\mathbf{k}-\mathbf{q}} \tag{3}$$

where

$$g_{\mathbf{q_o}} = \frac{\hbar D}{8N^2 mR\left(\rho_o + \frac{DR}{c^2}\right)} \frac{\mathbf{q}\mathbf{q}'}{\omega_{\mathbf{q}}\omega_{\mathbf{q_o}}} \sum_n e^{i\mathbf{q_o}\mathbf{z_n}} \tag{4}$$

D is a coupling constant, $\mathbf{q},\mathbf{q}'$ are the wave vectors associated with the bosons of the connecting field, $\mathbf{q}_o$ is the wave vector associated with the oscillations of the electron, and $\mathbf{k},\mathbf{k}'$ are the wave vectors of the electrons. $\omega_{\mathbf{q}},\omega_{\mathbf{q}_o}$ are the classical oscillation frequencies, $a_{\mathbf{q}}^{+}$ and $a_{\mathbf{q}}$ are the boson creation and annihilation operators, respectively, $c_{\mathbf{k}\sigma}^{+}$ and $c_{\mathbf{k}\sigma}$ are the creation and annihilation operators for electrons, $\boldsymbol{k}$ is the wave vector of an electron, $n_{\mathbf{q}}$ is the occupation number for bosons and $n_{\mathbf{k}}$ is the occupation number for fermions. We assume $n_{\mathbf{q}},n_{\mathbf{q}_o}=0,n_{\mathbf{k}},n_{\mathbf{k}-\mathbf{q}}=1$.

If instead of the Fröhlich fraction

$$\frac{\omega_{\mathbf{q}}}{\left(\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}-\mathbf{q}}\right)^2-\omega_{\mathbf{q}}^2} \qquad (5a)$$

we use the fraction

$$\frac{-1}{\left(\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}-\mathbf{q}}\right)-\omega_{\mathbf{q}}} \qquad (5b)$$

then for $\rho_o=0$ (mass less interacting field), (2) becomes

$$E_I=\frac{\hbar^3c^4}{32m^2R^4}\sum_{\mathbf{q},\mathbf{q}_o,\mathbf{k}}\frac{\left(\mathbf{q}.\mathbf{q}_o\right)^2}{\omega_{\mathbf{q}}^2\omega_{\mathbf{q}o}^2}\left|\sum_n e^{i\mathbf{q}_o.\mathbf{R}_n}\right|^2\frac{-1}{\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}-\mathbf{q}}-\omega_{\mathbf{q}}} \qquad (2a)$$

Now we apply this equation to a system of two electrons at $\mathbf{R}_1$ and $\mathbf{R}_2$ acting on the vacuum of the less mass boson field. In this case

$\sum_n\left|e^{i\mathbf{q}_o.\mathbf{z}_n}\right|^2=2\left(1+\cos(\mathbf{q}_o.\mathbf{R})\right),\ \varepsilon_{\mathbf{k}}=\hbar k^2/2m,$ and

$\omega_{\mathbf{q}}=cq,\omega_{\mathbf{q}_o}=\hbar q_o^2/2m,$

$\mathbf{R}=\mathbf{R}_1-\mathbf{R}_2$.

Further, if we assume $\left(\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}-\mathbf{q}}\right)<<\omega_{\mathbf{q}}$, we write

$$\sum_{\mathbf{q}}\frac{\left(\mathbf{q}.\mathbf{q}_o\right)^2}{\omega_{\mathbf{q}}^3\omega_{\mathbf{q}_o}^2}=\left(\frac{2m}{\hbar}\right)^2\frac{1}{q_o^2c^3}\frac{\Omega}{(2\pi)^2}\int_0^{\pi}\cos^2\alpha\sin\alpha d\alpha\times$$

$$\int_0^{q_o}qdq=\left(\frac{2m}{\hbar}\right)^2\frac{R^3}{9\pi c^3}$$

and

$$2\sum_{\mathbf{q}_o}\left[1+\cos\left(\mathbf{q}_o.\mathbf{R}\right)\right]=2\sum_{\mathbf{q}_o}1+2\sum_{\mathbf{q}_o}\cos\left(\mathbf{q}_o.\mathbf{R}\right)=$$

$$2+2\frac{\Omega}{(2\pi)^2}\int_0^{0.76\pi/R}dq_oq_o^2\int_0^{\pi}d\theta\cos\left(q_oR\cos\theta\right)\sin\theta=3.028$$

We have considered N = 1, $\sum_{\mathbf{k}}1=1$ and $\Omega=4\pi R^3/3$.

The upper limit of the integrals over q,q_o appear from the requirement

$$\frac{4\pi R^3/3}{(2\pi)^3}\times4\pi\frac{q_m^3}{3}=1$$

The interaction energy (2*a*) becomes

$$E_I\approx2\times7.24\times10^{-3}\frac{\hbar c}{R}\approx2\times\frac{1}{137}\frac{\hbar c}{R}=2\frac{e^2}{4\pi\varepsilon_oR}$$

The factor 2 appears because we have considered the two nearest neighbours of an electron. The interaction energy between two electrons is

$$E_I=\alpha\frac{\hbar c}{R} \qquad (6)$$

where $\alpha=1/137$ is the fine structure constant. This is an equivalent expression for the Coulomb's law. Expression (6) is obtained from (2*a*), containing the fraction (5*b*), which is valid for the interaction between the like charges acting in a mass less boson field. In this case the flux lines of the two particles do not interfere, the two particles absorb bosons from the ambient space, and move apart to one another. In the case where the two particles have opposite charges, then in a mass less boson field, the interaction energy is given by (2*a*) where the fraction (5b) is substituted by fraction (5*a*) and the interaction energy (6) becomes negative (in front off the term from the right hand side is a negative sign). The explanation for this is as follows. Fröhlich obtained the fraction (5*a*) by dividing fraction (5*b*) into two parts

$$\frac{1}{2}\frac{1}{\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}-\mathbf{q}}-\omega_{\mathbf{q}}}-\frac{1}{2}\frac{1}{\varepsilon_{\mathbf{k}'}+\omega_{\mathbf{q}}-\varepsilon_{\mathbf{k}'+\mathbf{q}}}$$

one part for absorption and the other for emission of a boson. When the two charges have opposite sign, there is a continuity of the field lines from one particle to the other, one particle absorbs and the other emits a boson, so that an attraction is assured. Also, an attraction may be assured between the like charges, when the connecting field is a massive field; likewise, in this case the interaction energy is given by (2).

When a particle has a charge *Q*, that is Q/e electronic charges, the term from the right hand side of Eq. (6) is multiplied by Q_1Q_2/e^2, because in this case we must define $|\Psi|^2 = Q/e$ (the number of electronic charges per particle).

2. LAMB SHIFT IN THE THREE DIMENSIONAL SPACE.

From equations (3) and (4) one obtains

$$E_{I2} = \frac{\hbar^3 D^2}{16m^2R^2\left(\rho_o + \frac{DR}{c^2}\right)^2}\frac{4\pi V}{(2\pi)^3}\int_{mc/\hbar}^{\infty} q_o^2 dq_o \frac{4\pi V}{(2\pi)^3}\times$$

$$\int_0^{q_o} q^2 dq \frac{q^2 q_o^2}{\omega_q^2 \omega_{q_o}^2}\sum_{\nu'}\frac{1}{\varepsilon_{\nu'} - \varepsilon_\nu - \omega_{\mathbf{q}}} \tag{7a}$$

We recognize that there is also a shift for free states. For free electrons (57*a*) becomes

$$E_{I2f} = \frac{\hbar^3 D^2}{8m^2R^2\left(\rho_o + \frac{DR}{c^2}\right)^2}\frac{4\pi V}{(2\pi)^3}\int_{mc/\hbar}^{\infty} q_o^2 dq_o \frac{4\pi V}{(2\pi)^3}\times$$

$$\int_0^{q_o} q^2 dq \frac{q^2 q_o^2}{\omega_q^2 \omega_{q_o}^2}\frac{-1}{\omega_{\mathbf{q}}} \tag{7b}$$

One gets the physical energy shift by substracting expression (7*b*) from (7*a*). The expression renormalized in this way is

$$\delta E = E_{I2} - E_{I2f} = \frac{\hbar^3 D^2 V^2}{64\pi^4 m^2 R^2\left(\rho_o + \frac{DR}{c^2}\right)^2}\int_{mc/\infty}^{\infty} q_o^2 dq_o \times$$

$$\int_0^{q_o} q^2 dq \frac{q^2 q_o^2}{\omega_{\mathbf{q}}^2 \omega_{\mathbf{q_o}}^2}\sum_{\nu'}\left(\frac{1}{\varepsilon_{\nu'} - \varepsilon_\nu - \omega_{\mathbf{q}}} + \frac{1}{\omega_{\mathbf{q}}}\right) =$$

$$\frac{\hbar^3 D^2\left(4\pi R^3/3\right)^2}{64\pi^4 m^2 R^2\left(\rho_o + \frac{DR}{c^2}\right)^2}\int_{mc/\hbar}^{\infty} q_o^2 dq_o \int_0^{q_o} q^2 dq \times$$

$$\frac{q^2 q_o^2}{\omega_{\mathbf{q}}^3 \omega_{\mathbf{q_o}}^2}\sum_{\nu'}\frac{\varepsilon_{\nu'} - \varepsilon_\nu}{\varepsilon_{\nu'} - \varepsilon_\nu - \omega_{\mathbf{q}}} \tag{8}$$

Now we assume $\rho_o = 0 \quad \omega_{\mathbf{q}} = cq, \omega_{\mathbf{q_o}} = \hbar q_o^2/2m_{\mathbf{q}} = \text{cq}$, and $\varepsilon_{\nu'} - \varepsilon_\nu << \omega_{\mathbf{q}}$. Therefore

$$\delta E = -\frac{\hbar R^2}{9\pi^2}\int_{mc/\hbar}^{\infty} dq_o \int_0^{q_o} dq \sum_{\nu'}\left(\varepsilon_{\nu'} - \varepsilon_\nu\right) \tag{9}$$

The expectation values of the energy $\hbar\varepsilon_\nu = E_\nu$ are determined from the Schrödinger equations

$$H_o\Psi_\nu = \left(-\hbar^2\frac{\nabla^2}{2m} - \frac{Z\alpha\hbar c}{r}\right)\Psi_\nu = E_\nu\Psi_\nu$$

$$H\Psi_{\nu'} = \left[-\hbar^2\frac{\nabla^2}{2m} - Z\alpha\hbar c\left(\frac{1}{r} - \frac{1}{2}(\delta r)^2\nabla^2\left(\frac{1}{r}\right)\right)\right]\Psi_{\nu'} =$$

$$E_{\nu'}\Psi_{\nu'} \tag{10}$$

Ψ_ν is the nonrelativistic wave function in the hydrogen like atom. $(\delta r)^2 = s^2$, where s^2 is the mean square value of s_l (Dolocan, 2005;2010)

$$s^2 = \frac{1}{R^2}\frac{\hbar^2}{4m^2\omega_{\mathbf{q}_o}^2}(2n_{\mathbf{q}_o} + 1) \tag{11}$$

Even in the lowest state ($n_{\mathbf{qo}} = 0$) the oscillator has a finite amplitude with a finite probability. In this case, $s^2 = \left(1/R^2q_o^4\right)$. By using that

$$\nabla^2\left(\frac{1}{r}\right) = -4\pi\delta(\mathbf{r})$$

and by using Eqs. (6) may be written

$$\sum_{\nu'}\left(\varepsilon_{\nu'} - \varepsilon_\nu\right) = -2\times\frac{1}{2}Z\alpha\ c\frac{1}{R^2q_o^4}\int\Psi_{\nu l}^*(\mathbf{r})\times$$

$$\nabla^2\left(\frac{1}{r}\right)\Psi_{\nu l}(\mathbf{r})d\mathbf{r} = -\frac{4\pi Z\alpha c}{R^2q_o^4}\left|\Psi_{\nu l}(0)\right|^2\delta_{l0} =$$

$$-\frac{Z\alpha c}{2a_o^5q_o^4}\delta_{l0} \tag{12}$$

A factor 2 appears because of the two values of the electron spin. For p orbitals, the nonrelativistic wave function vanishes at the origin, so there is no energy shift. But for s orbitals there is some finite value at the origin

$$\Psi_{2s}(0) = \left(\frac{1}{8\pi a_o^3}\right)^{1/2}$$

where we have denoted $R \equiv a_o$, the Bohr radius. By substituting (12) in (9) we have

$$\delta E_{2o} = \frac{Z\alpha\ \hbar c}{18\pi^2 a_o^3} \int_{mc/\hbar}^{\infty} \frac{dq_o}{q_o^4} \int_0^{q_o} dq = \frac{Z\alpha\ \hbar^3}{36\pi^2 m^2 c a_o^3} \tag{13}$$

For hydrogen atom, Z = 1, this shift is about 6.42×10^{-25} J which correspond to a frequency of 970 MHz. Further we consider the contribution to the Lamb shift of the interaction terms from the fine-structure Hamiltonian in according to Dirac theory (Bransden and Joachain, 1983). The first term of interaction is the usual spin-orbit coupling

$$H_2' = \frac{1}{2m^2c^2}\frac{1}{r}\frac{dV}{dr}\mathbf{L}.\mathbf{S} = \frac{1}{2m^2c^2}\frac{Z\alpha\hbar c}{r^3}\times \frac{1}{2}\left(\mathbf{J^2}-\mathbf{L^2}-\mathbf{S^2}\right) \tag{14}$$

and the second term of interaction is the Darwin term due to the nonlocalized interaction between the electron and the field

$$H_3' = \frac{\hbar^2}{8m^2c^2}\nabla^2 V = -\frac{\hbar^2}{8m^2c^2}\frac{2Z\alpha\hbar c}{r^3} \tag{15}$$

In the case of the spin-orbit coupling

$$\sum_{\nu'}\left(\varepsilon_{\nu'}-\varepsilon_{\nu}\right) = 2\times\frac{1}{2m^2c^2}\frac{Z\alpha\ \hbar^2 c s^2}{2}\int \Psi^*_{\nu l j m_j}(\mathbf{r})\times \nabla^2\left(\frac{1}{r^3}\right)\mathbf{LS}\Psi_{\nu l j m_j}\,d\mathbf{r} = \frac{6Z\alpha\ \hbar^2 s^2}{m^2 c}\times \left[\,j(j+1)-l(l+1)-\frac{3}{4}\right]\int \Psi^*{}_{\nu l}(\mathbf{r})\frac{1}{r^5}\Psi_{\nu l}(\mathbf{r})d\mathbf{r} = \frac{6Z\alpha\ \hbar^2 s^2}{m^2 c}\left[\,j(j+1)-l(l+1)-\frac{3}{4}\right]\left\langle\frac{1}{r^5}\right\rangle \tag{16}$$

By substituting (16) in (9) one obtains

$$\delta E_{\mathbf{LS}}^{21} = -\frac{Z\hbar^3\alpha}{3\pi^2 m^2 c}\frac{\hbar^2}{m^2c^2}\left\langle\frac{1}{r^5}\right\rangle_{21}\times\begin{cases}1 & \text{for j}=1+1/2\\ -2 & \text{for j}=1\text{-}1/2\end{cases} \tag{17}$$

By using the radial wave function of the electron in the hydrogen atom

$$R_{21}(r) = \frac{1}{\sqrt{3}}\left(\frac{Z}{2a_o}\right)^{3/2}\left(\frac{Zr}{a_o}\right)\exp\left(-Zr/a_o\right)$$

and

$$\left\langle\frac{1}{r^5}\right\rangle_{21} = \int_{\lambda_C}^{\infty}\left|R_{21}\right|^2\frac{dr}{r^3}$$

one obtains

$$\left\langle\frac{1}{r^5}\right\rangle_{21} = \frac{0.15}{a_o^5}$$

and $\delta E_{\mathbf{LS}}^{21} = 1.2\times10^{-28}$ J, which correspond to a frequency of ~ 0.18 MHz. In the above expression $\lambda_C = h/mc$ is the Compton wave length. The spin-orbit coupling contribution is zero for s electrons. Next we consider the Darwin term (15). We write

$$\hbar\sum_{\nu'}\left(\varepsilon_{\nu'}-\varepsilon_{\nu}\right) = -2\times\frac{\hbar^2}{8m^2c^2}\frac{1}{2}\alpha\ s^2\hbar c\times \int\Psi^*_{\nu l}(\mathbf{r})\nabla^4\left(\frac{1}{r}\right)\Psi_{\nu l}(\mathbf{r}) = \frac{3\hbar^3\alpha}{m^2c}\frac{1}{R^2q_o^4}\left\langle\frac{1}{r^5}\right\rangle \tag{18}$$

and

$$\delta E_{21}^{Darwin} = \frac{\hbar^3\alpha}{3\pi^2m^2c}\left\langle\frac{1}{r^5}\right\rangle_{21}\int_{mc/\hbar}^{\infty}\frac{dq_o}{q_o^5}\int_0^{q_o}dq = \frac{\hbar^3\alpha}{6\pi^2m^2c}\frac{0.15}{a_o^5}\frac{\hbar^2}{m^2c^2} = 2.9\times10^{-29}\,\text{J} \tag{19}$$

The Darwin contribution to the 2s level is

$$\delta E_{20}^{Darwin} = \frac{\hbar^3\alpha}{3\pi^2m^2c}\frac{\hbar^2}{m^2c^2}\left\langle\frac{1}{r^5}\right\rangle_{20} \tag{20}$$

By using the radial wave function of the electron in the hydrogen atom

$$R_{20} = 2\left(\frac{1}{2a_o}\right)^{3/2}\left(1-\frac{r}{2a_o}\right)\exp(-r/2a_o)$$

one obtains

$$\left\langle\frac{1}{r^5}\right\rangle_{20} = \int_{h/mc}^{\infty}\left|R_{20}\right|^2\frac{dr}{r^3} = \frac{101.392}{a_o^5}$$

and $\delta E_{20}^{Darwin} = 2\times 10^{-26}$ J. In the last equations we have considered $Z = 1$. The contribution of the Darwin term is of the order of 60.2 MHz. By adding the Darwin contribution to the contribution (13) one obtains $\nu = 1030.2$ MHz , which is close to the experimental value of the Lamb shift.

3.THE FINE STRUCTURE CONSTANT IN THE TWO DIMENSIONAL SPACE.

In the two dimensional space may be written

$$\left|\sum_l e^{i\mathbf{q_o}.\mathbf{R}_l}\right|^2 \delta_{R_{12},R_o} = 2\left[1+\frac{1}{2\pi}\int_0^{2\pi} e^{iq_oR_o\cos\theta}d\theta\right] = 2\left[1+J_o(q_oR_o)\right] \qquad (21)$$

where $J_o(x)$ is the Bessel function of the first kind, $\mathbf{R}_{12} = \mathbf{R}_1 - \mathbf{R}_2$. Further,

$$\sum_{\mathbf{q}}\frac{(\mathbf{q}.\mathbf{q_o})^2}{\omega_{\mathbf{q}}^2\omega_{\mathbf{q}_o}^2} = \left(\frac{2\mu}{\hbar}\right)^2\sum_q\frac{q^2q_o^2\cos^2\alpha}{q_o^4q^3c^3}\left(\frac{2\mu}{\hbar}\right)^2\times$$

$$\sum_q\frac{\cos^2\alpha}{q_o^2qc^3} = \frac{S}{(2\pi)^2}\left(\frac{2\mu}{\hbar}\right)^2\frac{1}{c^3q_o^2}\int_0^{2\pi}\cos^2\alpha d\alpha\int_0^{q_o}dq =$$

$$\frac{\pi R^2}{(2\pi)^2}\left(\frac{2\mu}{\hbar}\right)^2\frac{\pi}{c^3q_o} = \left(\frac{2\mu}{\hbar}\right)^2\frac{R^2}{4c^3q_o} \qquad (22a)$$

Next,

$$\sum_{\mathbf{q},\mathbf{q}_o}\frac{(\mathbf{q}.\mathbf{q}_o)^2}{\omega_{\mathbf{q}}^3\omega_{\mathbf{q}_o}^2}\left[1+J_o(q_oR_o\right] = \left(\frac{2\mu}{\hbar}\right)^2\frac{R^2}{4c^3}\frac{S}{(2\pi)^2}\times$$

$$2\pi\int_0^{2/R}\frac{1+J_o(q_oR_o)}{q_o}q_odq_o =$$

$$\left(\frac{2\mu}{\hbar}\right)^2\frac{R^2}{4c^3}\frac{\pi R^2}{2\pi}\frac{1}{R}\left[2+\int_0^2 J_o(q_oR_o)d(q_oR_o)\right] =$$

$$\left(\frac{2\mu}{\hbar}\right)^2\frac{R^3}{8c^3}\left[2+1.426\right] \qquad (22b)$$

The interaction energy in the two dimensional space becomes

$$E_I^{2D} = -\frac{\hbar^3c^4}{16\mu^2R^4}\frac{4\mu^2}{\hbar^2}\frac{R^3}{8c^3}\times 3.426 = -2\times 0.053\frac{\hbar c}{R} = -2\times\frac{\alpha_{2D}\hbar c}{R} \qquad (23)$$

where the fine structure constant in the two dimensional space is

$$\alpha_{2D} = 0.053 = 7.26\alpha \qquad (23a)$$

where α is the fine structure constant in the three dimensional space. It appears that in the two dimensional space the Coulomb interaction ia approximately 7 times stronger than in the three dimensional space. Here $1/\mu = 1/M_p + 1/m$, where μ is the reduced mass of the proton and electron in the hydrogen atom. We consider $\mu \approx m$, the electron mass. The electron radius in the hydrogen atom in the two dimensional space is

$$a_n^{2D} = \frac{\hbar}{\alpha_{2D}\mu c}\left(n-\frac{1}{2}\right)^2 \qquad (24)$$

where *n* is the principal quantum number. The Bohr radius a_1^{2D} is 29 times larger than the Bohr radius in the three dimensional space. The increasing of binding energy of the electron in the hydrogen atom is

$$\frac{E_n^{2D}}{E_n^{3D}} = \frac{\alpha_{2D}^2}{\alpha^2}\frac{n^2}{\left(n-\frac{1}{2}\right)^2} \qquad (25)$$

For the ground state, the binding energy in the two dimensional space is 200 times larger than that in the three dimensional space. It results that if should be a two dimensional space, in this space the matter should be more condensate than in the three dimensional space. We specify that the two dimensional hydrogen atom was studied in the past by many authors(Zaslow and Zandler, 1967; Yang et al, 1991; Guo et al, 1991; Taut, 1995).

4. LAMB SHIFT IN THE TWO DIMENSIONAL SPACE.

In the two dimensional space the expression (8) for the Lamb shift becomes

$$\delta E = \frac{\hbar^3D^2}{16m^2R^2\left(\rho_o+\frac{DR}{c^2}\right)^2}\frac{S^2}{(2\pi)^2}\int_{mc/\hbar}^{\infty}q_odq_o\times \int_0^{q_o}qdq\frac{q^2q_o^2}{\omega_{\mathbf{q}}^2\omega_{\mathbf{q}_o}^2}\sum_{\nu'}\frac{\varepsilon_{\nu'}-\varepsilon_\nu}{(\varepsilon_{\nu'}-\varepsilon_\nu)-\omega_{\mathbf{q}}} \qquad (26)$$

No, we assume $\rho_o = 0, \omega_{\mathbf{q}} = cq, \omega_{\mathbf{q}_o} = \hbar q_o^2/2m, \varepsilon_{\nu'}-\varepsilon_\nu << \omega_{\mathbf{q}}$, so that

$$\delta E = -\frac{\hbar}{16}\int_{mc/\hbar}^{\infty}\frac{dq_o}{q_o}\int_{mc/\hbar}^{q_o}\frac{dq}{q}\sum_{\nu'}\left(\varepsilon_{\nu'}-\varepsilon_{\nu}\right)$$

(26*a*)

We write

$$\sum_{\nu'}\left(\varepsilon_{\nu'}-\varepsilon_{\nu}\right)=2\times\frac{1}{2}Z\alpha_{2D}c\frac{1}{R^2q_o^4}\times$$

$$\int\Psi_{\nu l}^{*}(\mathbf{r})\nabla^2\left(\frac{1}{r}\right)\Psi_{\nu l}(\mathbf{r})d\mathbf{r}=\frac{2Z\alpha_{2D}c}{R^2q_o^4}\left\langle\frac{1}{r^3}\right\rangle_{\nu l}$$

For the 3D case, since $\nabla^2V(\mathbf{r})=-4\pi\delta(\mathbf{r})$, δE is nonzero only for the *s* electron. However this is no longer true for the 2D hydrogen atom, this term is nonzero for all the electrons. By using that

$$\nabla^2\left(\frac{1}{r}\right)=\frac{2}{r^3}:\quad\Psi_{2o}(\mathbf{r})=\frac{\beta_2}{\sqrt{3}}e^{-\beta_2 r/2}(1-\beta_2 r)\times$$

$$\frac{1}{\sqrt{\pi}}e^{i|m|\phi}=R_{2o}(\mathbf{r})\frac{e^{i|m|\phi}}{\sqrt{2\pi}}$$

$$\Psi_{21}(\mathbf{r})=\frac{\beta_2^2 r}{\sqrt{6}}e^{-\beta_2 r/2}\frac{e^{i|m\phi|}}{\sqrt{2\pi}}=R_{21}(\mathbf{r})\frac{e^{i|m|\phi}}{\sqrt{2\pi}};$$

$$\beta_2=\frac{4}{3}\frac{mc}{\hbar}\alpha_{2D}$$

(27)

For 2s level we have

$$\left\langle\frac{1}{r^3}\right\rangle_{20}=\frac{\beta_2^2}{3}\times 0.091\beta_2=0.03\beta_2^3$$

and therefore

$$\delta E_{2s}^{2D}=\frac{\hbar}{16}\frac{2Z\alpha_{2D}c}{\left(a_2^{2D}\right)^2}\times 0.03\beta_2^2\int_{mc/\hbar}^{\infty}\frac{dq}{q}\int_{mc/\hbar}^{q}\frac{dq_o}{q_o^5}=$$

$$\frac{3.75\times10^{-3}}{4}\frac{\alpha_{2D}\hbar c}{\left(a_2^{2D}\right)^2}\beta_2^3\left\{\int_{mc/\hbar}^{\infty}\frac{dq}{q^5}-\left(\frac{\hbar}{mc}\right)^4\int_{mc/\hbar}^{2/R}\frac{dq}{q}\right\}=$$

$$4.38\times10^{-4}\alpha_{2D}^6mc^2\left\{\frac{1}{4}-\ln\left(\frac{8}{9}\alpha_{2D}\right)\right\}=1.98\times10^{-25}\,\mathrm{J}$$

(28)

which correspond to a frequency of 300 MHz. For 2p level

$$\left\langle\frac{1}{r^5}\right\rangle_{21}=\frac{0.193}{6}\beta_2^5$$

and $\delta E_{2p}^{2D}=5.28\times10^{-5}\,\mathrm{J}$ which corresponds to a frequency of ~800 MHz.

For the spin-orbit coupling we have

$$\sum_{\nu'}\left(\varepsilon_{\nu'}-\varepsilon_{\nu}\right)=2\frac{\hbar}{m^2c^2}\frac{\alpha_{2D}\hbar c}{\left(a_2^{2D}\right)^2}\times$$

$$\left[j(j+1)-l(l+1)-3/4\right]\left\langle\frac{1}{r^5}\right\rangle_{21}\qquad(29)$$

and

$$\delta E_{\mathbf{LS}}^{2p}=0.0065\alpha_{2D}^8mc^2=3.31\times10^{-26}\,\mathrm{J}\qquad(30)$$

which corresponds to a frequency of 50 MHz.

For the Darwin term we have

$$\hbar\sum_{\nu'}\left(\varepsilon_{\nu'}-\varepsilon_{\nu}\right)=\frac{3\hbar^3\alpha_{2D}}{m^2c}\frac{1}{R^2q_o^4}\left\langle\frac{1}{r^5}\right\rangle\qquad(31)$$

and

$$\delta E_{2D}^{Darwin}=\frac{\hbar^5\alpha_{2D}^3}{27m^4c^3}\left\langle\frac{1}{r^5}\right\rangle\qquad(32)$$

so that

$$\delta E_{2D,2s}^{Darwin}=8.27\times10^{-4}\alpha_{2D}^8mc^2=4.22\times10^{-27}\,\mathrm{J}$$

which corresponds to a frequency of 6.3 MHz, and

$$\delta E_{2D,2p}^{Darwin}=5\times10^{-3}\alpha_{2D}^8mc^2=2.55\times10^{-26}\,\mathrm{J}$$

which corresponds to a frequency of 38 MHz. The total shift in the two dimensional space is of the order of 500 MHz. This value of the shift is some smaller than that in the three dimensional space.

5. CONCLUSIONS.

We have presented a theory of the Lamb shift without taken into account the electron charge. This appears as a natural result of the equivalent expression for the Coulomb's interaction energy, $\alpha\hbar c/R$, just derived from our Hamiltonian of interaction. This gives rise to the questions: may be the electron taken off the charge, or the electron is an indestructible charge? What is this the charge? We specify that in 1947 Hans Bethe was the first to explain the Lamb shift in the hydrogen spectrum, and be thus laid the foundation for the modern development of quantum electrodynamics. Neither the mass nor the charge of the electron or any other charged particle can actually calculated in QED- they have to be assumed. In our theory it is not necessary to assume a priori the charge of the electron. Also,

we have found that in the two dimensional space, the matter is more condensate than in the three dimensional space. In another paper, we extend these results to the interaction between nucleons via mass less bosons and massive particles (Dolocan et al, to be published).

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