Application of Ranking Function to solve Fuzzy Location-Routing problem

Azizollah Jafari*. Farhaneh Golozari**

**University of Science and Culture, Tehran, Iran (Tel: +989126869772; e-mail: Jafari@use.ac.ir).*
*** University of Science and Culture, Tehran, Iran (e-mail: f.golozari@use.ac.ir)*

Abstract: This paper addresses the Fuzzy Location-Routing problem (FLRP), raised by distribution networks involving depot locations, customer assignment and routing decisions in fuzzy environment. The FLRP is formulated as fuzzy linear programming (FLP) with fuzzy coefficients of the objective function and a fuzzy maximum travelling time constraint. Also this paper develops a simple approach to solve FLRP with travelling times along particular roads being L-R fuzzy numbers. The idea is based on the FLP and fuzzy number ranking method.

Keywords: supply chain management; fuzzy location-routing; ranking function; fuzzy linear programming; L-R fuzzy numbers.

1. INTRODUCTION

A key driver of the overall productivity and profitability of a supply chain is its distribution network which can be used to achieve a variety of the supply chain objectives ranging from low cost to high responsiveness. Designing a distribution network consists of three sub problems: location-allocation problem, vehicle routing problem, and inventory control problem. Because of high dependency among these problems, in the literature there are several papers integrating two of the above problems: location-routing problem (LRP), inventory-routing problem, and location-inventory problem (Ahmadi Javid and Azad, 2010). Although there are lots of researches in all of the three sub problems, but in this paper we focus on only LRP.

The LRP aims at simultaneously determining the location and routing decisions. Min et al., (1998) proposed a classification of LRP based on depot capacities, on the kind of vehicle fleet (homogeneous or heterogeneous), and on the costs of vehicles. Nagy and Salhi, (2007) proposed a more recent survey. Mathematical models with two-index or with three-index flow formulations were introduced (Laporte, 1988) but exact solutions methods are still limited to medium-scale or to basic uncapacitated instances. Numerous heuristic and meta-heuristic approaches for capacitated depots or for capacitated vehicles (but not both) were proposed. More recently, some authors published on the LRP with capacities on both depots and routes, called capacitated LRP (CLRP), e.g. Wu et al, 2002, Prins et al., (2007). Moreover, some authors published on the LRP with more constraints and condition such as maximum travelling time or maximum distance constraint on vehicles, e.g. Lin and kwok, (2006), Salhi and Nagy, (2007), Caballero et al., (2007), stochastic demands of customers, e.g. Chan et al., (2001), and different mathematical model, e.g. Albareda-sambola et al., (2005) and Aksen and Altinkemer, (2008). Also in this paper, we consider a CLRP in fuzzy environment with maximum travelling time constraint on vehicles.

In real-world environment, the travelling time along a particular road changes with the period of the day owing to predictable events such as congestion during peak hours or unpredictable events such as accidents, vehicle breakdowns Ichoua et al., (2003). Because of these events, the information about travelling time along a particular road is often not precise enough. For example, based on experience, it can be concluded that the travelling time along a particular road is "around 5 hours", "between 4 and 6 hours". Generally, it could be possible to use fuzzy numbers to deal with these uncertain parameters. According to the literature there is no paper which uses fuzzy data in LRP. Caballero et al., (2007) also mentioned application of fuzzy information in LRP as future work. Zheng and Liu in 2006 and Teodorovic and Kikuchi in 1991 applying fuzzy numbers for the travelling time along a particular road. Also Brito et al., (2009) reviewed fuzzy vehicle routing problem and different fuzzy approaches to solve fuzzy linear programming with fuzzy demands, fuzzy travelling time and etc. So in this paper, we consider FLRP with travelling time along a particular road as L-R fuzzy number.

The reminder of the paper is organized as follows. In section 2, mathematical formulation of the problem is given. Section 3 presents the solution method for solving the proposed model. In section 4, a numerical example solved with the method. We conclude the paper in section 5.

2. PROBLEM DESCRIPTION AND FORMULATION

The goal of our model is to choose, locate and allocate a set of depots, to schedule vehicles' routes to meet customers' demands in fuzzy environment such that the total cost is minimized. We assume that each customer has a certain demand. In the model, we consider different capacity levels for each depot which makes the problem more realistic and increases the capacity utilization of depots to a high level. Our assumptions and decisions determined by the model are explained as follows.

2.1 Assumption

- The demand of each customer must be satisfied and fulfilled from a depot.
- Each customer is served by exactly one vehicle.
- The total demand on each route is less than or equal to the capacity of a vehicle assigned to that route, and also
- The total travel time which a vehicle spent on each route is less than or equal to the maximum travelling time.
- Each route begins and ends at the same depot.
- The vehicles all have the same capacity and the same maximum travelling time constraint.

2.2 Decisions

- Location, capacity level and allocation decisions: how many depots to locate, where to locate the opened depots, what capacity level to consider for each of them, and how to allocate the customers to them.
- Routing decisions: how to build the vehicles' routes starting from an opened depot to serve its allocated customers according to both capacity and maximum travelling time constraint in fuzzy environment.

Now we integrate these two decisions in a mathematical programming model under the aforementioned assumptions. Before presenting the model, let us introduce the notation used throughout the paper.

2.3 Index sets

I : set of all potential depot site
J : set of all customers
K : set of all vehicles

2.4 Parameters and notations

N : Number of customers
$\tilde{d}_{ij}$: Travelling time between points i and j
G_i : fixed costs of establishing depot i
F : fixed cost of using vehicle k
V_i : Maximum throughput at depot i
dem_j : Demand of customer j
Q : Capacity of vehicle (route) k
T : The maximum time allowed for vehicles
λ : Variable transportation cost
β : Weight factor associated with transportation cost
θ : Weight factor associated with dispatching cost

2.5 Decision variables

x_{ijk} = If point i immediately proceeds point j on route k is equal to 1, otherwise is equal to 0.

y_i = If depot i is established is equal to 1, otherwise is equal to 0.

z_{ij} = If customer j is allocated to depot i is equal to 1, otherwise it is equal to 0.

2.5 Mathematical model

The problem formulation is as follows.
Min

$$Z=\sum_{i\in I}G_i y_i+\beta\sum_{i\in I\cup J}\sum_{j\in I\cup J}\sum_{k\in K}\lambda\tilde{d}_{ij}x_{ijk}+\theta\sum_{k\in K}F\sum_{i\in I}\sum_{j\in J}x_{ijk} \tag{1}$$

Subject to:

$$\sum_{k\in K}\sum_{i\in I\cup J}x_{ijk}=1, \qquad j\in J \tag{2}$$

$$\sum_{j\in J}dem_j\sum_{i\in I\cup J}x_{ijk}\le Q \qquad k\in K \tag{3}$$

$$U_{lk}-U_{jk}+Nx_{ijk}\le N-1 \qquad l,j\in J,\ k\in K \tag{4}$$

$$\sum_{j\in I\cup J}x_{ijk}-\sum_{j\in I\cup J}x_{jik}=0 \qquad k\in K,\ i\in I\cup J \tag{5}$$

$$\sum_{i\in I}\sum_{j\in J}x_{ijk}\le 1 \qquad k\in K \tag{6}$$

$$\sum_{j\in J}d_j z_{ij}-V_i y_i\le 1 \qquad i\in I \tag{7}$$

$$-z_{ij}+\sum_{u\in I\cup J}\left(x_{iuk}+x_{ujk}\right)\le 1 \qquad i\in I,\ j\in J,k\in K \tag{8}$$

$$\sum_{i\in I\cup J}\sum_{j\in I\cup J}\tilde{d}_{ij}x_{ijk}\le T \qquad k\in K \tag{9}$$

$$x_{ijk}\in\{0,1\} \tag{10}$$

$$z_{ij}\in\{0,1\} \tag{11}$$

$$y_i\in\{0,1\} \tag{12}$$

$$U_{lk}\ge 0 \tag{13}$$

$$0<\beta,\theta\le 1 \tag{14}$$

The objective function in (1) minimizes the sum of the fixed depot establishing cost, transportation cost and dispatching cost for vehicles assigned, respectively. Equation (2) requires that each customer be assigned to a single route. Equation (3) is the capacity constraint set for vehicles. Equation (4) is the sub-tour elimination constraint. Flow conservation constraints are expressed in (5). Equation (6) assures that each route can be served at most once. Capacity constraints for depots are given in (7). Equation (8) specifies that a customer can be assigned to a depot only if there is a route from that depot going through that customer. Equation (9) express maximum travel time limit. Equations (10), (11) and (12) are the binary requirements on the decision variables. The auxiliary variables taking positive values are declared in (13). Acceptable amount of weight factors is shown in (4).
As you can see in the objective function, only the parameters which related to second part of objective function are fuzzy numbers and just this part is an imprecise objective function. By applying the strategy (Kaufmann and Gupta, 1991) that a nonfuzzy numbers such as r can be expressed as a triangular

fuzzy number, (r,r,r), we converted all parameters in the first and third parts of objective function into the fuzzy numbers. You can see it as follow.

$$G_i = (G_i, G_i, G_i)$$

$$F = (F, F, F)$$

By replacing these fuzzy numbers into the objective function, we have

Min (15)

$$\tilde{Z} \cong \sum_{i\in I} \tilde{G}_i y_i + \beta \sum_{i\in I\cup J} \sum_{j\in I\cup J} \sum_{k\in K} \lambda \tilde{d}_{ij} x_{ijk} + \theta \sum_{k\in K} \tilde{F} \sum_{i\in I} \sum_{j\in J} x_{ijk}$$

Now we have a completely fuzzy (imprecise) objective function which replaced by the first objective function. We also use this strategy to convert the maximum travelling time, T into a fuzzy number, (T,T,T) and use it in maximum travelling time constraint (9). It is shown as follow.

$$\sum_{i\in I\cup J} \sum_{j\in I\cup J} \tilde{d}_{ij} x_{ijk} \le (T,T,T)$$

$$\sum_{i\in I\cup J} \sum_{j\in I\cup J} \tilde{d}_{ij} x_{ijk} \le \tilde{T} \qquad (16)$$

In the next section, we introduce an algorithm based on ranking function to solve FLRP.

3. THE SOLUTION PROCEDURE

To deal with this problem, one approach which has been proved to be correct is to transform the fuzzy numbers to crisp ones. This paper transforms FLRP model to crisp one via defuzzifying the fuzzy parameters in the objective function (15), and in the maximum travelling time constraint, (16), by using a fuzzy ranking method which is simple and the conventional crisp linear programming (LP) solvers can still workable. This paper adopts Yager's ranking method for ranking the objective values and maximum travelling time constraint. Before presenting the idea of this paper, we briefly introduce the Yager ranking function.

3.1 Yager ranking function

Yager in 1997 proposed a procedure for ordering fuzzy sets, in which a ranking index $I(\tilde{t})$ is calculated for the convex fuzzy number $\tilde{t}$ from it's $\alpha - cut$, $t_\alpha = [t_\alpha^L, t_\alpha^U]$ according to the following formula:

$$I(\tilde{t}) = \int_0^1 \frac{1}{2}(t_\alpha^L, t_\alpha^U) d\alpha \qquad (17)$$

Which is the centre of the mean vale of $\tilde{t}$. Considering two fuzzy numbers $\tilde{D}_1$ and $\tilde{D}_2$, the case of $I(\tilde{D}_1) \ge I(\tilde{D}_2)$ implies that $\tilde{D}_1 \ge \tilde{D}_2$, and $\max\{\tilde{D}_1, \tilde{D}_2\} = \tilde{D}_1$ (Yager, 1997, Fortemps and Roubens, 1996). This function is very simple to apply, and according to (17), since it is calculated for the convex fuzzy numbers $\tilde{t}$ from the extreme values of its $\alpha - cut$, t_α^L and t_α^U, rather than its membership function, it is not require knowing the explicit form of the membership functions of the fuzzy numbers to be ranked. That is unlike most of the ranking methods that require the knowledge the membership functions of all fuzzy numbers to be ranked, the Yager's ranking function is still applicable even if the explicit form of the membership function of fuzzy numbers is unknown.

3.2 Crisp transformation

Consider the fuzzy objective function which is modelled in (15). By applying Yager's function to find the best objective function, we have

$$I(\tilde{z}) \cong I(\sum_{i\in I} \tilde{G}_i y_i + \beta \sum_{i\in I\cup J} \sum_{j\in I\cup J} \sum_{k\in K} \lambda \tilde{d}_{ij} x_{ijk} +$$

$$\theta \sum_{k\in K} \tilde{F} \sum_{i\in I} \sum_{j\in J} x_{ijk}) = I(\sum_{i\in I} \tilde{G}_i y_i) + I(\sum_{i\in I\cup J} \sum_{j\in I\cup J} \sum_{k\in K} \lambda \tilde{d}_{ij} x_{ijk})$$

$$+ I(\theta \sum_{k\in K} \tilde{F} \sum_{i\in I} \sum_{j\in J} x_{ijk}) = \sum_{i\in I} I(\tilde{G}_i) y_i + \sum_{i\in I\cup J} \sum_{j\in I\cup j} \sum_{k\in K} \lambda I(\tilde{d}_{ij}) x_{ijk}$$

$$+ \theta \sum_{k\in K} I(\tilde{F}) \sum_{i\in I} \sum_{j\in J} x_{ijk}$$

And also by applying this method to fuzzy maximum travelling constraint, we have

$$I(\sum_{i\in I\cup J} \sum_{j\in I\cup J} \tilde{d}_{ij} x_{ijk}) \le I(\tilde{T})$$

$$\sum_{i\in I\cup J} \sum_{j\in I\cup j} I(\tilde{d}_{ij}) \le I(\tilde{T})$$

In next section, we solve a numerical example by this method.

4. NUMERICAL EXAMPLE

In this section, to illustrate the efficiency of the proposed approach, a numerical is solved.

Let us consider a FLRP with maximum travelling time constraint. We assume that there are 6 customers labelled "1, 2,3,4,5, 6" and 3 potential depots labelled "7, 8, and 9". We also assume that the travelling time along a particular road (between customers, between customers and depots) is fuzzy numbers. These travelling times are fuzzy numbers of $L_{ij} - R_{ij}$ type, $(i,j) \in I \cup J$ (Zimmerman, 2001). The notation used in this paper is $\tilde{d} = (m, \overline{m}, a, b)$ for an L-R fuzzy number whose the membership function is as follows:

$$\mu_{\tilde{d}}(d) = \begin{cases} L\left(\frac{m-d}{a}\right), & d \le m \\ 1, & m \le d \le \overline{m} \\ R\left(\frac{d-\overline{m}}{b}\right), & d \ge \overline{m} \end{cases} \qquad (18)$$

Where a and b are nonnegative real numbers, and L and R are called reference functions of this fuzzy numbers, which are continuous, nonincreasing functions that defining the left and right shapes of $\mu_{\tilde{d}}(d)$, respectively; and $L(0) = R(0) = 1$.

Five commonly used nonlinear references functions with parameter $q\ (q \geq 1)$, denoted as RF_q, and are summarized as follows (Zimmerman, 2001):

Linear: $$RF(x) = \max(0, 1-x), \tag{19}$$

Exponential: $$RF_q(x) = e^{-qx}, \tag{20}$$

Power: $$RF_q(x) = \max(0, 1-x^q), \tag{21}$$

Exponential power: $$RF_q(x) = e^{-x^q}, \tag{22}$$

Rational: $$RF_q = \frac{1}{(1+x^q)}, \tag{23}$$

In this example, the L-R fuzzy travelling times are shown in table 1.

Table 1, Fuzzy travelling time matrix

	1	2	3	4	5	6	7	8	9
1	-	(1,2.5,1,1)	(0.5,2.5,0,2)	(1,2,2,4)	(0.5,2,2,2)	(1,3,2,0)	(3,6,2,3)	(1.5,3.5,2,2)	(1,3.5,2,2)
2	(1,2.5,1,1)	-	(1.5,4.5,2,3)	(0.5,2.5,1,2)	(1.5,3.5,1,2)	(2,4,1,1)	(2.5,3.5,2,0)	(0.5,2.5,1,2)	(2,6,1,1)
3	(0.5,2.5,0,2)	(1.5,4.5,2,3)	-	(0.5,2,2,0)	(1,2,2,3)	(2,3,0,2)	(4,6,2,4)	(1,2.5,1,1)	(1,4,2,2)
4	(1,2,2,4)	(0.5,2.5,1,2)	(0.5,2,2,0)	-	(1,2.5,2,0)	(1.5,2.5,2,4)	(3,5.5,0,2)	(1,2,2,0)	(0.5,4,0,2)
5	(0.5,2,2,2)	(1.5,3.5,1,2)	(1,2,2,3)	(1,2.5,2,0)	-	(1,2.5,2,3)	(0.5,2,1,1)	(2.5,6,0,2)	(5,6,1,1)
6	(1,3,2,0)	(2,4,1,1)	(2,3,0,2)	(1.5,2.5,2,4)	(1,2.5,2,3)	-	(3,5,2,2)	(1,4,2,4)	(1,2,2,0)
7	(3,6,2,3)	(2.5,3.5,2,0)	(4,6,2,4)	(3,5.5,0,2)	(0.5,2,1,1)	(3,5,2,2)	-	(4,6,1,2)	(4,5,2,2)
8	(1.5,3.5,2,2)	(0.5,2.5,1,2)	(1,2.5,1,1)	(1,2,2,0)	(2.5,6,0,2)	(1,4,2,4)	(4,6,1,2)	-	(3,4,2,3)
9	(1,3.5,2,2)	(2,6,1,1)	(1,4,2,2)	(0.5,4,0,2)	(5,6,1,1)	(1,2,2,0)	(4,5,2,2)	(3,4,2,3)	-

And the reference functions are

$$L_{12} = L_{21} = L_{15} = L_{51} = L_{18} = L_{81} = L_{24} = L_{42} = L_{27} = L_{72} = L_{34} = L_{43} = L_{37} = L_{73} = L_{45} = L_{54} = L_{48} = L_{84} = L_{57} = L_{75} = L_{67} = L_{76} = L_{98} = L_{89} = \max(0, 1-x)$$

$$L_{13} = L_{31} = L_{16} = L_{61} = L_{19} = L_{91} = L_{25} = L_{52} = L_{28} = L_{82} = L_{35} = L_{53} = L_{38} = L_{83} = L_{46} = L_{64} = L_{49} = L_{94} = L_{58} = L_{85} = L_{68} = L_{86} = L_{97} = L_{79} = e^{-2x}$$

$$L_{14} = L_{41} = L_{17} = L_{71} = L_{23} = L_{32} = L_{26} = L_{62} = L_{29} = L_{92} = L_{36} = L_{63} = L_{39} = L_{93} = L_{47} = L_{74} = L_{56} = L_{65} = L_{59} = L_{95} = L_{96} = L_{69} = L_{78} = L_{87} = \max(0, 1-x^2)$$

$$R_{12} = R_{21} = R_{15} = R_{51} = R_{18} = R_{81} = R_{24} = R_{42} = R_{27}$$
$$R_{72} = R_{13} = R_{31} = R_{16} = R_{61} = R_{19} = R_{91} = R_{25} = R_{52}$$
$$R_{14} = R_{41} = R_{17} = R_{71} = R_{32} = R_{23} = R_{26} = R_{62} = \max(1-x^2, 0)$$

$$R_{34} = R_{43} = R_{37} = R_{73} = R_{45} = R_{54} = R_{48} = R_{84} = R_{28} = R_{82} = R_{35} = R_{53} = R_{38} = R_{83} = R_{46} = R_{64} = R_{29} = R_{92} = R_{36} = R_{63} = R_{39} = R_{93} = R_{74} = R_{47} = e^{-2x}$$

$$R_{57} = R_{75} = R_{67} = R_{76} = R_{98} = R_{89} = R_{49} = R_{94} = R_{58}$$
$$R_{85} = R_{68} = R_{86} = R_{97} = R_{79} = R_{56} = R_{65} = R_{59} = R_{95}$$
$$R_{96} = R_{69} = R_{78} = R_{87} = \max(0, 1-x)$$

Demands of customers are given in Table 2. Fixed established costs of potential depots are given in Table 3. The variable transportation cost is equal to 70. All vehicles have the same capacity which equal to 70, also all of them has a same maximum travelling time which equal to 8. In this numerical example, we consider a same capacity for all potential depots which equal to 140.

Table 2, Customers' demand

Number of customer	1	2	3	4	5	6
demands	27	28	23	29	22	28

Table 3, Fixed established cost of depots

Number of depots	1	2	3
established cost	10841	11961	6091

As shown in (17), to use the Yager method, for calculating the ranking indices for the fuzzy travelling times, $\tilde{d}_{ij}$ firstly we have to find the $\alpha - cut\ (\alpha \in (0,1])$ of $\tilde{d}_{ij}$ which can be obtained by finding the inverse functions of these reference function:

For linear: $$RF^{-1}(\alpha) = 1 - \alpha \tag{24}$$

For exponential: $$RF_q^{-1}(\alpha) = -(\ln \alpha)/q \tag{25}$$

For power: $RF_q^{-1}(\alpha) = \sqrt[q]{1-\alpha}$ (26)

For exponential power: $RF_q^{-1}(\alpha) = \sqrt[q]{-\ln\alpha}$ (27)

For rational: $RF_q^{-1}(\alpha) = \sqrt[q]{(1-\alpha)/\alpha}$ (28)

The Yager's ranking indices for $\tilde{d}_{ij}$ are calculated as:

$$I(\tilde{d}_{14}) = I(\tilde{d}_{41}) = I(\tilde{d}_{17}) = I(\tilde{d}_{71}) = I(\tilde{d}_{32}) = I(\tilde{d}_{23}) = I(\tilde{d}_{26}) = I(\tilde{d}_{62}) = 0.67$$

$$I(\tilde{d}_{13}) = I(\tilde{d}_{31}) = I(\tilde{d}_{16}) = I(\tilde{d}_{61}) = I(\tilde{d}_{19}) = I(\tilde{d}_{91}) = I(\tilde{d}_{25}) = I(\tilde{d}_{52}) = I(\tilde{d}_{12}) = I(\tilde{d}_{21}) = I(\tilde{d}_{78}) = I(\tilde{d}_{87}) = I(\tilde{d}_{18}) = I(\tilde{d}_{81}) = I(\tilde{d}_{24}) = I(\tilde{d}_{42}) = I(\tilde{d}_{27}) = I(\tilde{d}_{72}) = I(\tilde{d}_{29}) = I(\tilde{d}_{92}) = I(\tilde{d}_{36}) = I(\tilde{d}_{63}) = I(\tilde{d}_{39}) = I(\tilde{d}_{93}) = I(\tilde{d}_{47}) = I(\tilde{d}_{74}) = I(\tilde{d}_{56}) = I(\tilde{d}_{65}) = I(\tilde{d}_{59}) = I(\tilde{d}_{95}) = I(\tilde{d}_{96}) = I(\tilde{d}_{69}) = 0.58$$

$$I(\tilde{d}_{34}) = I(\tilde{d}_{43}) = I(\tilde{d}_{37}) = I(\tilde{d}_{73}) = I(\tilde{d}_{45}) = I(\tilde{d}_{54}) = I(\tilde{d}_{48}) = I(\tilde{d}_{84}) = I(\tilde{d}_{57}) = I(\tilde{d}_{75}) = I(\tilde{d}_{67}) = I(\tilde{d}_{76}) = I(\tilde{d}_{89}) = I(\tilde{d}_{98}) = I(\tilde{d}_{28}) = I(\tilde{d}_{82}) = I(\tilde{d}_{35}) = I(\tilde{d}_{53}) = I(\tilde{d}_{38}) = I(\tilde{d}_{83}) = I(\tilde{d}_{46}) = I(\tilde{d}_{64}) = I(\tilde{d}_{49}) = I(\tilde{d}_{94}) = I(\tilde{d}_{58}) = I(\tilde{d}_{85}) = I(\tilde{d}_{68}) = I(\tilde{d}_{86}) = I(\tilde{d}_{97}) = I(\tilde{d}_{79}) = 0.5$$

Substituting these values into FLRP model which coding this model in LINGO 8, by running it on a personal computer with 2.80 GHz Pentium R processor and 3.11 GB of RAM, the objective function calculated 19032.96 in 24:02 minutes.

6. CONCLUSIONS

For the first time, in this paper, we introduce a fuzzy location-routing problem with fuzzy objective function and fuzzy maximum travelling time constraint in order to considering more condition in realistic world. This problem is formulated as fuzzy linear mathematical programming. Also we introduce an effective procedure to solve this model and a numerical example show the efficiency of the proposed method.

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