Speckle filtering in radar polarimetric images by wavelet transform in complex domain

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Abstract: Polarimetric SAR images are disturbed by an inherent noise called "speckle" having multiplicative properties. This noise is undesirable, and its treatment in the case of polarimetric SAR images is difficult because of its complicated modelling. To reduce its level of disturbance, a polarimetric filtering is necessary to improve the image quality, with preserving polarimetric information. In this paper, we present two wavelet filtering techniques based on multi-scale edge detection and wavelet thresholding by applying the stationary wavelet transform (SWT). The originality of our work was based on new approaches proposed in POLSAR images filtering by wavelets to filter polarimetric covariance matrix elements and complex channels individually. The methods are applied to the fully polarimetric (HH, HV, VH, VV) SAR images acquired on Algiers, Algeria and to three polarimetric channels (HH, HV, VV) SAR images acquired on Oberpfaffenhofen area located in Munich (Germany). We evaluate the performance of each filter by using these following criteria: smoothing homogeneous areas, edge preservation and polarimetric information preservation. Experimental results are included to illustrate the method.

Keywords: Polarimetric speckle filtering, POLSAR images, stationary wavelet transform SWT, multi-scale edge detection, wavelet thresholding.

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1. INTRODUCTION

As in mono-polarisation radar SAR images, the polarimetric SAR images (POLSAR) are affected by a granular noise, named Speckle (Goodman, 1985). It appeared like a pepper and salt noise. The effect of speckle on polarimetric estimation parameters has been prospected by Goodman in optic (1963 ; 1975 ; 1985). Until 1999, the research of speckle effect on polarimetric statistics parameters has been a wide imagery SAR prospection domain (Barakat 1985 ; Eom and Boerner 1991 ; Lee and al 1994a,b ; Lopes and al 1992 ; Murza 1978 ; Quegan and Rhodes, 1995; Sarabandi, 1992; Touzi and Lopes 1991; 1996; Touzi and al. 1999; Vachula and Barnes 1983)

Murza (1978), Vachula and Barnes (1983) were the first to use the Wishart distribution to describe the polarimetric parameters of SAR images. Since then, Wishart distribution has been widely used to estimate multilook speckle effects on polarimetric tools (Lopes and al. 1992 ; Lee and al,1994a ,1999; Touzi and al., 1999) , also used as algorithms base for segmentation and classification (Lee and al, 1994b ; 1999, Ferro-Famil and al, 2001 ; Beaulieu and Touzi 2004) and also for edges detection techniques (Shou and al, 2003).

Speckle filtering for polarimetric data was been a research topic for many years ago (Novak end Burl, 1990; Lee and al, 1991; Touzi and Lopes, 1994; Shou and Skriver, 2001). Touzi and Lopez (1994) were the first to show that the use of conventionnal mono-polarised filter did not preserve polarimetric information (Lee, 1981 ; Lee and al, 1994a ; Touzi 2002) and that the speckle filtering must be applied on all the elements of covariance matrix and not only on the elements of scattering matrix. Many filters giving a filtered covariance matrix, or filtered Muller matrix, or filtered coherence matrix were develloped (Touzi and Lopes, 1994 ; Lee and al 1994b ; Lee and Grues 2000, Shou and skriver 2001)

During the ten last years, Lee and al, in 2006 (Lee and al. 2006) introduced a novel concept in Polsar speckle filtering which allowed the preservation of scattering diffusion mechanism “dominant” of each filtred pixel. The results presented by Lee were been confirmed by Touzi (Touzi 2007). The works of Lopez-Martinez (Lopez and al 2005, Lopez-Martinez and Pottier 2007) has allowed evaluating the speckle influence of coherence matrix on the estimation of the used values H/A/Alpha decomposition. He also showed that noise was not only multiplicative for the non-diagonal elements of covariance matrix, but an additive noise was also present (Lopez-Martinez and Pottier 2007).

Another works have also emerged, based on the multiscale decomposition, such as those developed by Lopez Martinez and al. in 2005 (Lopez-Martinez and al 2005) and by Farage and Foucher in the following years (Farage 2007, Foucher and al 2006, Foucher 2007).

Different wavelet transform methods (Donoho, 1993) (Grgic and al., 2001) have shown good results in many applications to solve a variety of image processing problems as compression and filtering. In the field of POLSAR filtering, speckle reduction by stationary wavelet transform (SWT) (Farage, 2007) has shown its effectiveness in providing a good compromise between smoothing homogenous areas and edge preservation in heterogeneous areas. We have implemented two wavelet filtering techniques: multi-scale edge detection filtering using proposed approaches for wavelet coefficients improvement and wavelet thresholding filtering (hard and soft thresholding) with their enhanced versions. Our contribution is linked to the new approaches proposed in POLSAR images filtering by wavelets to filter polarimetric covariance matrix elements and complex channels individually. The main goal of these approaches is to detect edge regions and no-edge regions and to classify significant coefficients. Then apply a suitable image thresholding and modify the wavelet coefficients to obtain a better quality filtered image which satisfies the criteria requested. These methods are compared to filtered images of Lee filter (Lee and al., 1999) and are applied on POLSAR images of Oberpfaffenhofen located in Munich (Germany) (Bouchemakh, 2008) and on Algiers images (Algeria).

Statistical and visual evaluations for a comparative study are performed to validate the studied methods.

2. FILTERING BY WAVELET TRANSFORM

Wavelets are an effective tool for image processing applications. They can identify and analyze the discontinuities in the image at different levels. This property is used for filtering the wavelet coefficients before making the image reconstruction. In what follows, we first recall the principle of wavelet transform used, then we consider the principle of filtering by multi-scale detection and coefficients thresholding.

The covariance matrix C used in the following sections is defined by:

$$Vect = \begin{bmatrix} C_{11}, C_{22}, C_{33}, \mathrm{Re}(C_{12}), \mathrm{Re}(C_{13}), \mathrm{Re}(C_{23}), \mathrm{Im}(C_{12}), \\ \mathrm{Im}(C_{13}), \mathrm{Im}(C_{23}) \end{bmatrix} \quad \mathbf{(1)}$$

Where *Vect* is the input images vector, *Re* and *Im* are the real and imaginary part of the complex image, respectively.

2.1 Stationary Wavelet Transform (SWT)

The wavelet transform used in the filtering method is the stationary wavelet transform (SWT) (Farage and al., 2007). The SWT generates four images, three high frequency images called wavelet coefficients corresponding to the horizontal, vertical and diagonal directions noted by : W_h^j, W_v^j, W_d^j, representing the original image details and an low-frequency image called approximate image noted by A^j, bringing the highest percentage of information content among the four images. *T*he transformation generates an equal number of wavelet coefficients at all scales. j represents the number of scale (j = 1, ..., J). The transform SWT is similar to the discrete wavelet transform DWT (Discrete Wavelet Transform), except that the image is not decimated and in each level decomposition, the filters are up-sampled by inserting zeros between each filter coefficient. Then, details images are the same size as the original image.

2.2 Filtering by multi-scale edge detection

To provide robustness to speckle filtering, the amplitude of the operator at j is expressed as follows (Scharcanski, 2002):

$$M_n^j = \sqrt{\sum_{\varepsilon=h,v,d} \left(W_\varepsilon^j\right)^2} \quad \mathbf{(2)}$$

n is the number of the input image.

The procedure for classifying wavelet coefficients proposed by Farage (Farage and al., 2007) based on the SSC (Sum of Squared Coefficients) is given as follows:

$$g^j = \begin{cases} 1 & si \ \sum_{n=1}^{N} \left(M_n^j\right)^2 > T \\ 0 & otherwise \end{cases} \quad \mathbf{(3)}$$

Where N is the total number of input images and T is the estimated threshold.

The edge coefficients tend to become broader at higher scales while the noise becomes smaller. If the image structures produce very large wavelet coefficients that must be preserved (Foucher and al.,2006, Foucher, 2007), a threshold is imposed as:

$$g^j = \begin{cases} 1 & si \ ECM \ \left\{M_n^j\right\}^{n=1,2,3} > \sqrt{L+2} \\ 0 & otherwise \end{cases} \quad \mathbf{(4)}$$

Where g^j is the binary mask at j.

The Enhancement Factor Method (ECM) is the improvement wavelet coefficients method by using the PCA (Principal Component Analysis) or the SSC(Sum of Squared Coefficients) with:

$$SSC = \sum_{n=1}^{N} \left(M_n^j\right)^2 \quad \mathbf{(5)}$$

Another classification method of edge and non-edge coefficients is proposed by Dachasilaruk in 2008. It is given by:

$$g^j = \begin{cases} 1 & si\ (M^j)^2 > \sigma_v^j \\ 0 & otherwise \end{cases} \quad \textbf{(6)}$$

If $g^j = 1$, we have an edge and if $g^j = 0$, we have not any edges in the region. For the calculation of σ^j, we propose to add in the equation (7) a parameter γ which allows to control the filtering effect, therefore, to obtain a good compromise between smoothing homogeneous areas and edges preservation of. The expression of σ^j becomes:

$$\sigma_v^j = \gamma\ Median\left(\left|M^j\right|^2\right)/0.6745 \quad \textbf{(7)}$$

Once the masks are obtained, the wavelet coefficients are multiplied by the shrinkage function such as :

$$\left(W_\varepsilon^j\right)' = g^j \times W_\varepsilon^j \qquad \text{with} \qquad \varepsilon = \text{h, v, d} \quad \textbf{(8)}$$

Through equation 9, we obtain the new filtered coefficients that will be used in the inverse wavelet transform to obtain the filtered image.

2.3 Filtering by wavelet thresholding

The wavelet thresholding is a filtering approach widely used in image processing because of its simplicity. In its simplest form, this technique is to compare each coefficient with a threshold, if the coefficient is smaller than the threshold, we set to zero, otherwise it is kept or modified. The theoretical formalization of thresholding in the context of noise removal by thresholding wavelet coefficients is presented by Mallat (Mallat, 1989) and Donoho (Donoho, 1995). In 2008, Dachasilaruk studied wavelet thresholding on single channel SAR images (Dachasilaruk, 2008). In our paper, we propose to extend this technique and to apply it on the polarimetric SAR images, using the polarimetric information in the calculation of the threshold on the wavelet coefficients. There are two thresholding methods: hard thresholding and soft thresholding. In this both cases, the coefficients that are lower than a given threshold are set to zero.

2.3.1 Hard thresholding.

The hard thresholding is defined by the following equation (10), where the threshold value T is calculated from the wavelet coefficients W_ε^j.

$$W' = \begin{cases} W_\varepsilon^j & si\ \left|W_\varepsilon^j\right| \geq T \\ 0 & si\ \left|W_\varepsilon^j\right| < T \end{cases} \quad \textbf{(9)}$$

2.3.2 Soft thresholding.

In the soft thresholding, the amplitudes of coefficients that are above the threshold T, are reduced to a different value. The soft thresholding is defined by the following equation (Dachasilaruk, 2008):

$$W' = \begin{cases} \operatorname{sgn}\left(W_\varepsilon^j\right)\left(\left|W_\varepsilon^j\right| - T\right) & si\ \left|W_\varepsilon^j\right| \geq T \\ 0 & si\ \left|W_\varepsilon^j\right| < T \end{cases} = \begin{cases} W_\varepsilon^j - T & si\ W_\varepsilon^j \geq T \\ 0 & si\ \left|W_\varepsilon^j\right| < T \\ W_\varepsilon^j + T & si\ W_\varepsilon^j \leq -T \end{cases} \quad \textbf{(10)}$$

The hard thresholding has a disadvantage because of his sudden discontinuity: the estimates have greater variance and can be very sensitive to small changes in values. In practice, especially when the noise level is high, the hard thresholding presents sudden changes in the reconstructed image. Contrariwise, in the soft thresholding, the reconstructed image is often more smoothed. For this reason, the soft thresholding is generally preferred to hard thresholding. Several standard methods of threshold calculating in wavelet filtering have been suggested (Donoho, 1995). The choice of the threshold is an important issue, and a number of publications are devoted to calculating the threshold. Instead of applying the same threshold on all coefficients (in a sub-band) (Farage and al., 2007, Dachasilaruk, 2008), it would be preferable to estimate the appropriate threshold for each coefficient separately. Donoho (Donoho, 1995) proposed the threshold VisuShrink, also called universal threshold. It provides a visually adaptive smoothing using the calibrated parameter γ, which leads to an asymptotically optimal estimating in the minimax sense (minimizing the maximum error over all the n sampled signals). The threshold T is proportional to the noise variance and is given by the following expression:

$$T = \gamma\,\sigma\sqrt{2\log(n)} = \gamma\,\frac{Median\left(\left|W_\varepsilon^j\right|\right)}{0.6745}\sqrt{2\log(n_w)} \quad \textbf{(11)}$$

In literature, the universal threshold (VisuShrink) of Donoho is the most widely used due to its simplicity of calculation and good estimation.

2.3.3 Implemented Methods Algorithms.

A- Method 1: SSC(M) and PCA(M): The steps of the multi-scale detection filtering method are listed below:

- Apply the stationary wavelet transform.
- Improving the wavelet coefficients M^j using PCA or SSC on the *Span*, the diagonal elements, or all elements.
- Classify the edge coefficients and no-edge coefficients, using the masking (6)
- Modify the wavelet coefficients by multiplying them by the mask g^j (8).
- Apply the inverse wavelet transform to produce the filtered images.

B- Method 2: New approach of multi-scale detection: We propose a new approach by introducing instead the second stage of the previous method, an improvement on the coefficients W_{ε}^{j} by computing the new expression of the directional $M_{\varepsilon=h,v,d}^{j}$ proposed, defined by:

$$M_{\varepsilon=h,v,d}^{j} = \sqrt{\sum_{n=1}^{N} \left(W_{\varepsilon}^{j}\right)_n^2} \qquad \textbf{(12)}$$

And in order to obtain a single coefficient containing all the information on the directional coefficients, which we perform the thresholding step to calculate the global mask that we will use later on each coefficient.

C- Method 3: Filtering by wavelet thresholding : The steps of wavelet thresholding filtering method are listed below:

- Apply the stationary wavelet transform.
- Choose an appropriate threshold to the used images and calculated it using the equation (11).
- Apply the hard thresholding (9) or soft thresholding (10) on the coefficients.
- Apply the inverse wavelet transform to produce the filtered images.

D- Method 4 : Enhanced filtering by wavelet thresholding :

We propose to improve the technique of both thresholding soft and hard, using the SSC or the ACP on the coefficients M ^{j} of Span, on the diagonal, or all items. Therefore, we perform the same steps of the method 3, except that we replace the second step by our proposal. The proposed method is illustrated in the block diagram in figure 1.

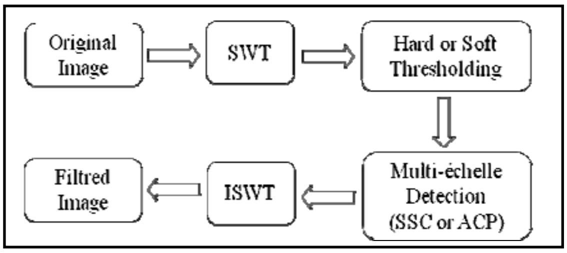

Fig. 1. Block diagram of the improved thresholding.

3. EXPERIMENTAL RESULTS

The filters are tested on extracted images from two single-look POLSAR complex images, one is an airborne one corresponding to the region of Oberpfaffenhofen in Germany acquired by ESAR sensor in 2001 (P-band) and the other corresponding to the area of Algiers in Algeria acquired by RADARSAT-2 in April 2009 (C-band). The evaluation of each filter is based on the following main criteria: Ability to smooth the homogeneous areas, ability to preserve edges and especially preserving the polarimetric information.

Ratings: Hard is the Hard wavelet thresholding filter, Soft is the soft wavelet thresholding filter, Ehard represents the improved Hard wavelet thresholding filter, Esoft represents the improved soft wavelet thresholding filter. SSC (Sum of Squared Coefficients) and PCA (Principle Component Analysis) are the Enhancement Coefficient Methods.

From the above figures, we see that the hard thresholding (Hard) shows a slight smoothing and a good preservation of fine details. The soft thresholding (Soft) presents a good homogeneity and preserve edges. The improved filters Ehard and Esoft have a strong smooth and satisfactory conservation structures. Thus, we conclude that the combination of the two thresholding methods with the multi-scale detecting offers obviously an image quality much better than in case of using simple thresholding Hard and Soft. Both filters SSC(W) and ACP(W) smooth homogeneous areas and preserve structures. In Munich images, the best filter with a very good smoothing of homogeneous areas is PCA (M) and the best filter that ensures proper preservation of linear structures is the SSC(M). In Algiers images, the PCA(M) filter presents a better smoothing textures and the Lee filter has a great ability to preserve edges but with slight fluctuations (nonlinearity) in the linear structures.

The visual evaluation is not sufficient to verify the required criteria and validate the studied filtering methods. Then we perform the following statistics calculations :

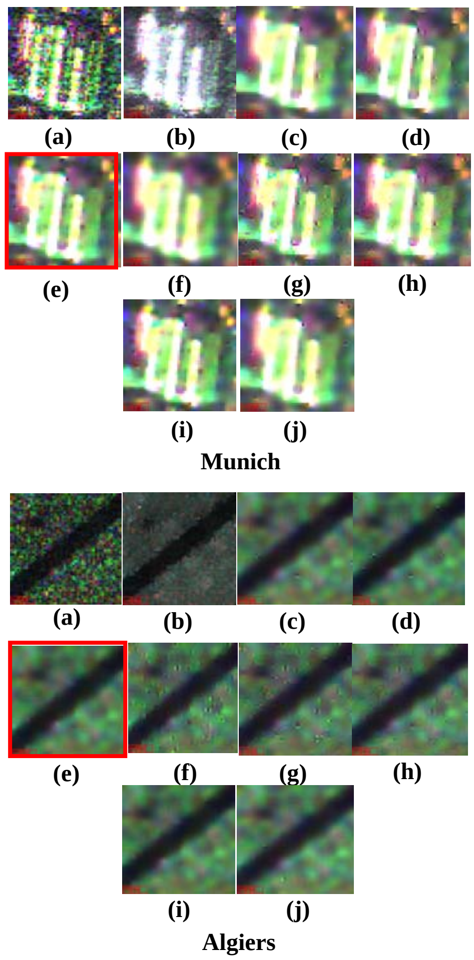

Fig. 2 : Intensities color composite zoom images of Munich and Algiers. **(a)** Original, **(b)** Lee, **(c)** SSC(W), **(d)** ACP(W), **(e)** SSC(M), **(f)** ACP(M), **(g)** Hard,**(h)** Soft, **(i)** Ehard, **(j)** Esoft.

Smoothing homogeneous areas:

A good filtering in homogeneous areas is provided by
- A low value of coefficient of variation Cv.
- Increased value of ENL.

From table 1, we note that the wavelet filters correspond to the required statistical criteria. The SSC (M), PCA (M) and the Ehard, Esoft thresholding offer ENL values higher than other filters. However, the SSC(M) method provides overall better results. The soft thresholding (Soft) also gave a ENL high value compared to hard thresholding and has a strong smoothing and low edges preserving. Overall, the statistical evaluation of the obtained results showed a great ability to preserve edges and smooth homogeneous areas. Thus, these results resonate with visual evaluation conclusions.

Edge preservation:

The best filter in terms of preservation of outlines is one that gives the largest coefficient of variation C_{VG}.

From table 2, we see that the best overall result in terms of preservation of contours in both cases intensity and complex filtering is given by the Lee filter flowed by wavelet filters treated with the SSC and PCA.

4. CONCLUSION

The comparative study between the filters was based on the visualization and statistical results of each method. The purpose of implemented filters is to have a homogeneous texture and good edge preservation. Thus, the filtered images have a good quality and are ready to be used in various applications such as classification, segmentation, creation of DEM (Digital Elevation Model),…etc.

Filtering by wavelet seems to give good results in POLSAR image processing, so it is possible to combine the wavelet coefficients thresholding to the reducing speckle procedure. Through the technical improvement coefficients, the wavelet thresholding and the new proposed approaches, the linear structures are little changed after filtering and smoothing is much more apparent. The undesirable effect of speckle is reduced, which improves image quality and facilitates interpretation for the identification of objects. The technique SSC(M) ensures a better compromise between a good smoothing of homogenous areas and good preservation of edges. This is due to its adaptivity. However, the choice of the threshold and the coefficient γ which ensures a good compromise between smoothing and preserving edges is not obvious.

Table 1. Statistical results in homogeneous areas

Munich			**Algiers**		
	Cv	**ENL**		**Cv**	**ENL**
Original	0.746	1.796	**Original**	0.638	2.454
Lee	0.460	4.719	**Lee**	0.267	13.962
SSC(W)	0.244	16.695	**SSC(W)**	0.147	45.872
SSC(M)	0.512	3.800	**SSC(M)**	0.181	30.406
Soft	0.220	20.595	**Soft**	0.147	45.659
Esoft(SSC)	0.247	16.308	**Esoft(SSC)**	0.147	45.872

Table 2. Statistical results in heterogeneous areas.

	Munich		*Algiers*	
Filters	***Intensity***	***Complex***	***Intensity***	***Complex***
Lee	1.744	1.744	4.583	4.583
SSC(M)	1.145	1.021	1.185	1.403
ACP(M)	0.870	1.399	1.211	1.308
Hard	1.208	1.218	1.086	1.113
Ehard(ACP)	0.870	1.194	1.211	1.367

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