Schrödinger Equation in Fractional Space

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Abstract:

The Schrödinger equation is studied in a fractional space. Specifically, we present the model of a hydrogen-like fractional atom called "fractional Bohr atom". The total Hamiltonian of the systems is given in terms of the fractional kinetic energy $C_\alpha p^\alpha$, $1 < \alpha \leq 2$ and the fractional potential energy $V(\mathbf{r}) = -\frac{Ze^2 k_\varepsilon}{|\mathbf{r}|^{\varepsilon+1}}$, where C_α is a constant, p is the linear momenta, Z is the atomic number, e is the charge of the electron, $0 < \varepsilon \leq 1$, and k_ε is a space constant given as $k_\varepsilon = \frac{\Gamma(\frac{\varepsilon+3}{2})}{2\pi^{(\varepsilon+3)/2}(\varepsilon+1)\epsilon_0}$. We present the virial theorem for a system with a general fractional potential and the kinetic energy of the above type, and then obtain expressions for the radius of the nth electron orbit and the frequency of radiation when an electron jumps from one energy level to another. It is shown that for $\varepsilon = 0$ and ($\alpha = 2$, $\varepsilon = 0$), the results obtained here agree with those presented by Laskin [2] and with those obtained using the classical quantum theory.

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[2] N. Laskin, arXiv: quant-phy/0206098.