Optimal Tuning Method for Fractional PI Controller Based on Diffusive Representation

B. Boudjehem . [*,**,1] **Dj. Boudjehem** [*] **H. Tebbikh** [*]

[*] *Laboratoire d'Automatique et Informatique de Guelma (LAIG), Université de Guelma, BP 401 Guelma, 24000, Algèrie*
[**] *Université de Skikda, Route d'El-Hadaiek BP 26 Skikda 21000,Algèrie*

Abstract: This paper propose an optimal tuning method for fractional PI controller. The method consist of minimizing Integral Time Absolute Error (ITAE) performance index criterion. The fractional PI controller are achieved by diffusive representation and the optimum setting of fractional controller parameters are reached by using conventional optimization algorithm. An example of application is presented to evaluate the proposed method. A comparison with classical PI controller shows that the system is robust to gain variations.

Keywords: Fractional-order system, fractional controller, performance index ITAE, optimization, diffusive representation

1. INTRODUCTION

Recently the concept of fractional calculus are widely introduced in many areas in science and engineering. In control systems, this concept are successfully used to construct fractional order controllers. As a result, the closed loop control system performances are improved in comparison with classical controllers.

In Podlubny (1999) proposed a generalization of the PID controller namely fractional PID ($PI^\lambda D^\mu$) where λ and μ are the order of integration and derivation respectively that can be real numbers. In comparison with classical PID these controllers have two extra parameters. Therefore classical design method may not be applied directly to adjust all fractional controller parameters.

Several research works have proposed new design techniques and tuning rules, for fractional controllers . Some of them are based on an extension of the classical control theory. In Valério and da Costa (2006) a tuning method for fractional PID controller based on Ziegler-Nichols-type rules was proposed. Monje et al. (2004) present a frequency domain approach based on the expected crossover frequency and phase margin. A state-space tuning method based on pole placement was also used (see Dorcak et al. (2001)). Recent tuning method based on Quantitative Feedback Theory (QFT) are presented in Natarj and Tharewal (2007).

Many methods for control design are based on optimization techniques. The common approach is to minimize a performance index (Aström and Hägglund (1995)). An optimization approach was proposed in Monje et al. (2004), for the PI fractional controller tuning. A nonlinear functional minimization subject to some given nonlinear constraints are solved using matlab minimization function. An intelligent optimization method for designing fractional order PID controller based on Particle Swarm Optimization (PSO) was presented (see Cao and Cao (2006)). In Leu et al. (2002), an optimal fractional order PID controller based on specified gain and phase margins with a minimum ISE criterion has been designed by using a differential evolutionary algorithm. Tuning fractional PID controller based on ITAE criterion by using Particle Swarm Optimization has been also presented in Maiti et al. (2008). In Tavazoei (2009) the infiniteness and finiteness of different performance indices in class of fractional-order systems have been presented.

In this paper we propose a tuning method for fractional PI controller based on minimizing integral time absolute error by means of diffusive representation. The feedback control system is implemented in Matlab/Simulink. Simulation results show the effectiveness of the proposed design method in comparison with classical PI controller. The paper is organized as follows. In Section 2 gives an overview on fractional order controllers and the diffusive representation. Section 3 presents the design method procedure. An illustrative example are given in section 4 to demonstrate the effectiveness of the proposed method. Finally, a conclusions are stated in section 5.

2. FRACTIONAL ORDER OPERATORS AND CONTROLLERS

2.1 Fractional order operator

There are several different definitions of fractional operators (see Oldham and Spanier (1974) and Miler (1993)). One of the most used definition of the fractional integration is the Riemann-Liouville definition

[1] Corresponding author: Dr. Boudjehem Badreddine E-mail: b_boudjehem@yahoo.fr

$$D^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(\tau) d\tau \tag{1}$$

while the fractional derivative definition is

$$D^{\beta} f(t) = D^m [D^{-\gamma} f(t)] \tag{2}$$

where

$$\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx \tag{3}$$

is the Gamma function, α is the order of the integration, m is an integer number and $\gamma = m - \beta$.

The Laplace transform method is a powerful tool in the frequency domain for both the system analysis and the controller synthesis. The Laplace transform of the fractional integral given by Riemann-Liouville, under zero initial conditions for order α is :

$$L\left(D^{-\alpha} f(t)\right) = s^{-\alpha} F(s) \tag{4}$$

where $F(s)$ is the normal Laplace transformation $f(t)$.

2.2 Fractional PID controller ($PI^{\lambda}D^{\mu}$)

Fractional $PI^{\lambda}D^{\mu}$ controller is a system described by a fractional differential equation

$$K_p \left(y(t) + \frac{1}{T_i} D^{-\lambda} y(t) + T_d D^{\mu} y(t) \right) = e(t) \tag{5}$$

where D is the derivative operation, K_p is the proportionally gain, T_i is the integration constant, T_d is the derivative constant, λ is the integration order and μ is the derivative order. The Laplace transform of (5), lead to the following transfer function

$$C(s) = K_p \left(1 + \frac{1}{T_i} s^{-\lambda} + T_d s^{\mu} \right) \tag{6}$$

Taking $\mu = 0$ and/or $T_d = 0$ we obtain a fractional PI. We note that if $\mu = 1$ and $\lambda = 1$, we obtain a classical PID controller.

2.3 Diffusive representation of fractional operators

There are several approaches that have been used to implement fractional order integration (see Point and Trigeassou (2002) and Oustaloup et al. (2000)). An alternative is to use the so-called "Diffusive approach" (see Montseny (2004)).

The diffusive realization of the pseudo differential operator H, with impulse response h, $u \to g = H\left({}^{d}/_{dt}\right) u$ is defined by the dynamic input-output system:

$$\begin{cases} \partial_t \varphi(\xi, t) = -\xi \varphi(\xi, t) + u(t) \\ g(t) = \int_0^{\infty} \mu(\xi) \varphi(\xi, t) \, d\xi \\ \varphi(\xi, 0) = 0, \quad \xi \succ 0 \end{cases} \tag{7}$$

The system (7) is the diffusive realization of H .

The impulse response $h(t)$is expressed from *h(t)* by:

$$h(t) = \int_0^{+\infty} e^{-\xi t} \mu(\xi) \, d\xi \tag{8}$$

so the diffusive symbol is also given by: $\mu = L^{-1} h$

The transfer function of the operator H is given by:

$$H(s) = \int_{-\infty}^{+\infty} \frac{\mu(\xi)}{s + \xi} d\xi \tag{9}$$

We thus have the three equivalent representations :

Diffusive rep. $\stackrel{L}{\to}$	Convolution rep. $\stackrel{L}{\to}$	symbol
μ	*h(t)*	*H(s)*
$\mu \# \nu$	*h(t)∗r(t)*	*H(s).R(s)*

In the particular case of fractional integrators

$$H\left(\frac{d}{dt}\right) = \left(\frac{d}{dt}\right)^{-\alpha}, 0 < \alpha < 1 \tag{10}$$

The diffusive symbol is expressed as (see Laudebat et al. (2004)):

$\mu(\xi) = \frac{\sin(\pi\alpha)}{\pi} \frac{1}{\xi^{\alpha}}$, x>0 where a is the order of integration

The numerical approximation of the fractional order system based on diffusive representation is simple and presents a more advantages.

3. THE PROPOSED DESIGN METHOD

Let us consider the feedback control system depicted in figure 1. Where $G(s)$ is the controlled system transfer function and $C(s)$ is the controller transfer function. The controller used is a fractional PI controller with transfer function given by (11).

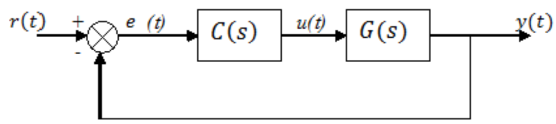

Fig. 1. Feedback control system

$$C(s) = K_p \left(1 + \frac{1}{T_i s^{\lambda}}\right) \tag{11}$$

These controllers have three unknown parameters K_p, T_i, and λ, that must be determined to achieved the desired specifications. Using (7), the diffusive realization of the controller with input e and output u is given by

$$\begin{cases} \partial_t \varphi(\xi, t) = -\xi \varphi(\xi, t) + e(t) \\ u(t) = \int_0^{\infty} \nu(\xi) \varphi(\xi, t) \, d\xi \end{cases} \tag{12}$$

where the diffusive symbol is:

$$\nu(\xi) = K_p \left(\delta(\xi) + \frac{1}{T_i} \frac{\sin(\pi\lambda)}{\pi} \frac{1}{\xi^{\lambda}} \right) \tag{13}$$

Thus the transfer function of the fractional controller PI^{λ} by means of diffusive representation are

$$C(s) = \int_{-\infty}^{+\infty} \frac{\nu(\xi)}{s+\xi} d\xi \qquad (14)$$

where ν is defined by (13)

Our objectives are to adjust the three fractional PI parameters (K_p, T_i, and λ) that minimizing the integral time absolute error ITAE defined by the objective function

$$J_{ITAE}(K_p, T_i, \lambda) = \int_0^T t|e(t)|dt \qquad (15)$$

where t is the time and $e(t)$ is the error step set-point change.

The procedures to determine the PI^{λ} fractional controller parameters are summarized in the following:

(1) Implement the feedback control system in Matlab/Simulink including diffusive realization of the fractional PI controller through Simulink model
(2) Calculate the ITAE error
(3) Use a function of Matlab optimization toolbox to minimize the objective function J. The initial controller parameters is set to be those determined by one of existing tuning rules.

4. AN EXEMPLE OF APPLICATION

In this section, an example of application is given to illustrate the proposed method. Consider the forth order system

$$G(s) = \frac{k}{s(s+1)(s+2)(s+3)} \qquad (16)$$

To illustrate the robustness to parameter variations, we consider only that the gain can be changed with a variation range of $K \in [1, 1.8]$. For the simulation the function of matlab optimization tool box are used to minimize the objective function J_{ITAE}.

The stability margin based Ziegler-Nichols is used for determine the initial parameters K_p and T_i whereas the order $\lambda = 1$. The optimized parameters of the fractional PI controller are :$K_p = 5.64$, $T_i = 76.32$, and $\lambda = 1.12$.

Therefore, the transfer function of the fractional controller is:

$$C(s) = 5.64(1 + \frac{1}{76.32 s^{1.12}}) \qquad (17)$$

Figure 2 presents the step response of the controlled system with fractional PI controller. This figure shows clearly that the overshoot and set time are acceptable.

Figure 3 illustrates the step responses with fractional controller for different values of K. This figure shows that the fractional controller designed by the proposed method permits to have a time responses with slightly iso-overshoot for different values of gain K.

In order to prove the efficiency of the proposed method we compare our results with those obtained using classical PI

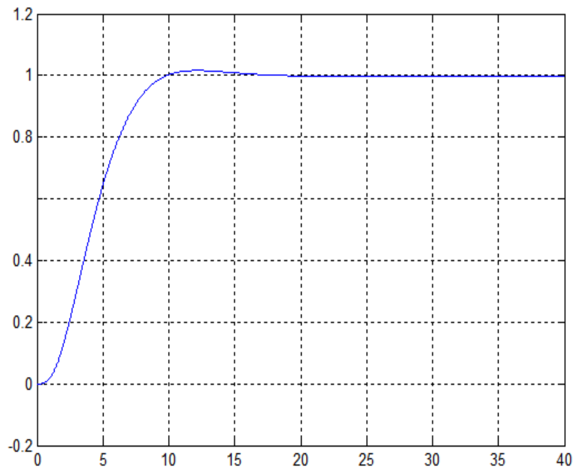

Fig. 2. Step response of the controlled system with fractional PI controller

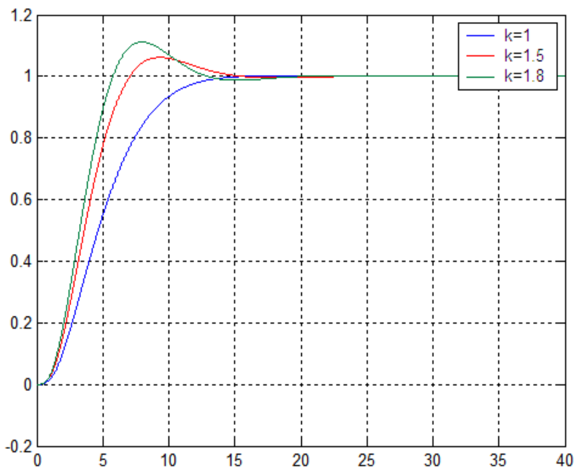

Fig. 3. Step response of the controlled system with fractional PI controller for different values of K

controller, the same procedure are applied to tune classical PI controller Therefore the optimal transfer function of classical PI controller

$$C(s) = 2.0512(1 + \frac{1}{3.0681e6s}) \qquad (18)$$

Figure 4 presents the step response of the controlled system with classical PI controller. Figure 5 illustrates the step responses with classical controller for different values of K. This figure shows that the responses with this controller present different overshoot. So that the system is not robust to gain variations.

5. CONCLUSIONS

An optimal design method for fractional PI controller has been presented. The method is based on using diffusive representation of fractional operator. The optimal settings have been obtained by minimizing Integral Time Absolute error using Matlab optimization toolbox . The simulation results have shown the effectiveness of the proposed

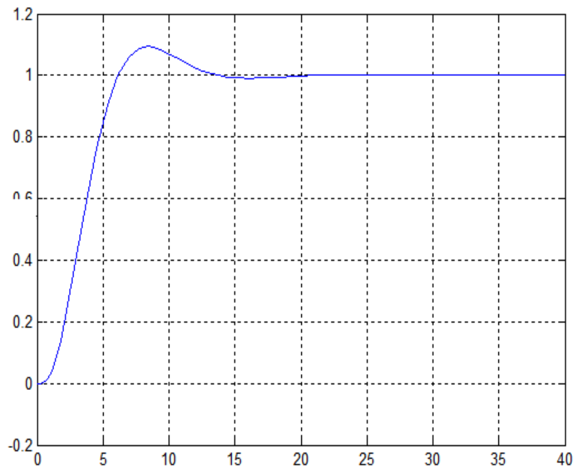

Fig. 4. Step response of the controlled system with classical PI controller

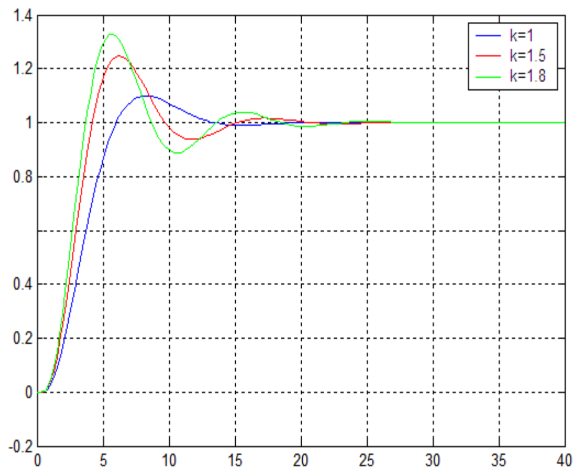

Fig. 5. Step response of the controlled system with classical PI controller for different values of K

method in comparison with classical PI controller. In addition the system obtain is robust to gain variations.

REFERENCES

K. Aström and T. Hägglund. *PID controllers:Theory, design, and tuning*. Instrument Society of America, North Carolina, 1995.

J. Cao and B.G. Cao. Design of fractional order controller based on particle swarm optimisation. *International journal of Control Automation, and systems*, 4:775–781, 2006.

L. Dorcak, I. Petras, I. Kostial, and J. Terpak. State-space controller desing for the fractional-order regulated system. *International carpathian control conference*, 1: 2001, 2001.

L. Laudebat, P. Bidan, and G. Montseny. Modeling and optimal identification of pseudodifferential dynamics by means of diffusive representationparti:modeling. *IEEE Transactions on Circuits and Systems*, 51:1801–1813, 2004.

F.J. Leu, S.Y. Tsay, and C. Hwang. Design of optimal fractional-order pid controllers. *Journal of the Chinese Institute of Chemical Engineers*, 33:193–202, 2002.

D. Maiti, A.A. Acharya, M. Chakraborty, A konar, and R. Janarthanan. Tuning pid and pi$^{\lambda}$d$^{\mu}$ controller using the integral time absolute error criteria. *International Conference on Information and Automation for Sustainability ICIAF8, Colombo, Sri lanka*, 2008.

S. Miler. *Tintroduction to the fractional calculus and fractional differential equations*. Wiley, NewYork, 1993.

C.A. Monje, A.J. Calderón, B.M. Vinagre, Y.Q. Chen, and V. Felieu. On fractional pi$^{\lambda}$ controllers: Some tuning rules for robustness to plant uncertainties. *Nonlinear Dynamics*, 8:369–381, 2004.

G. Montseny. Simple approach to approximation and dynamical realization of pseudodifferential time operators such as fractional ones. *IEEE Transactions on Circuits and Systems II*, 51:613–618, 2004.

P.S.V. Natarj and S. Tharewal. On fractional-order qft controllers. *Journal of Dynamic Systems, Measurement, and Control*, 129:212–218, 2007.

K. Oldham and J. Spanier. *The fractional calculus: Theory and application of differentiation and integration to arbitrary order*. Wiley, NewYork, 1974.

A. Oustaloup, F. Levron, B. Mathieu, and F. Nanot. Frequency-band complex non integer differentiator: Characterization and synthesis. *IEEE Transactions on Circuits and Systems I*, 47:25–39, 2000.

I. Podlubny. Fractional-order systems and pid-controllers. *IEEE Transactions on Automatic Control*, 44:208–213, 1999.

T. Point and J. C. Trigeassou. A method for modelling and simulation of fractional systems. *Signal Processing*, 83:2319–2333, 2002.

M. S. Tavazoei. Notes on integral performance indices in fractional-order control systems. *Journal of Process Control*, Article in press, 2009.

D. Valério and J. Sá da Costa. Tuning of fractional pid controllers with ziegler-nichols-type rules. *Signal processing*, 86:2771–2784, 2006.