Fractal Analysis to Quantify Wind Speed Fluctuations

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Instantaneous variation of wind speed due to changes in the weather can lead to electrical power variations. Such fluctuations can cause instabilities in the wind energy systems production. To overcome these difficulties, it is necessary to quantify wind speed fluctuations. In this paper we propose measuring these fluctuations using the fractal dimension. To estimate this parameter the *Rectangular Covering Method* we already developed is used. The method applied to hourly wind speed of Quebec station provides information that allows us to characterize the wind speed variation at this location

Keywords: Fractal dimension, rectangular covering method, Minkowski-Bouligand method, log-log plots, least squares estimation, wind speed, fluctuations, wind energy.

1. INTRODUCTION

A major issue in the control and stability of wind energy production systems is to maintain the balance between generated and consumed power. Because of the fluctuating nature of wind speeds, the use of wind turbines for power generation has caused more focus on the fluctuations in the power production of the wind turbines.

Indeed, instantaneous variation of wind speed due to changes in the weather can lead to electrical power variations. Lets recall that the wind power is dependent on the velocity of the wind to the third power, if the wind speed doubles, the power available from the wind increases by a factor of eight. Such fluctuations can cause instabilities in the wind energy systems production. Therefore, it is necessary to quantify the wind speed fluctuations.

This paper presents a fractal approach to measure the wind speed variations. The approach is based on the fractal dimension as a tool for measuring the degree of the wind speed curves irregularity.

Let us recall that the most significant characteristic of fractals is their fractal dimension. This latter which contains information about their geometrical structure is as a powerful tool for measuring the degree of their irregularity over multiple scales.

For a curve, fractal dimension is between 1 and 2, it approaches 2 if it is extremely irregular and tends towards 1 if it is more regular. Fractal dimension can then be used to compare the geometrical complexity of two curves.

2. FRACTAL DIMENSION ESTIMATION

To estimate the fractal dimension of curves, several methods exist. The most popular ones are the Box-Counting and the Minkowski-Bouligand methods. However, these techniques suffer from inaccuracy. Inspired by the Minkowski –Bouligand method, a class of approaches to compute the fractal dimension of signal curves or one-dimensional profiles called *covering methods* is then proposed by several researchers (Tricot *et al*., 1988; Dubuc *et al*., 1989; Maragos and Sun, 1993).

These methods consist in creating multiscale covers around the signal's graph. Indeed, each covering is formed by the union of specified structuring elements. In the method of Box–counting, the structuring element used is the square or limp, that of Minkowski –Bouligand uses the disk.

To improve the complexity and the precision of the fractal dimension estimation of time series, we have developed a new method called *Rectangular covering Method* (Harrouni *et al*., 2005; Harrouni and Guessoum, 2006, 2008) based on a rectangle as a structuring element of covering.

1.1 Rectangular Covering Method

The method consists in covering the curve for which we want to estimate fractal dimension by rectangles instead of disks like in Minkowski –Bouligand approach. The choice of this type of structuring element is due to the discrete character of the studied signals. Thus, the rectangle allows combining every point on the x-axis with the corresponding point on the y-axis, thus achieving the covering of the signal without information loss.

From the mathematical point of view, the use of the rectangle as structuring element for the covering is justified. Indeed, in (Bouligand, 1928) Bouligand showed that the Minkowski–Bouligand dimension can be obtained by also replacing the disks in the previous covers with any other arbitrarily shaped compact sets that posses a nonzero minimum and maximum distance from their centre to their boundary.

Thus, as shown in Fig. 1, for different time intervals $\Delta\tau$ the area $S(\Delta\tau)$ of this covered curve is calculated by using the following relation:

$$S(\Delta\tau)=\sum_{t_n=0}^{N-1}\Delta\tau\cdot\left|f(t_n+\Delta\tau)-f(t_n)\right| \qquad (1)$$

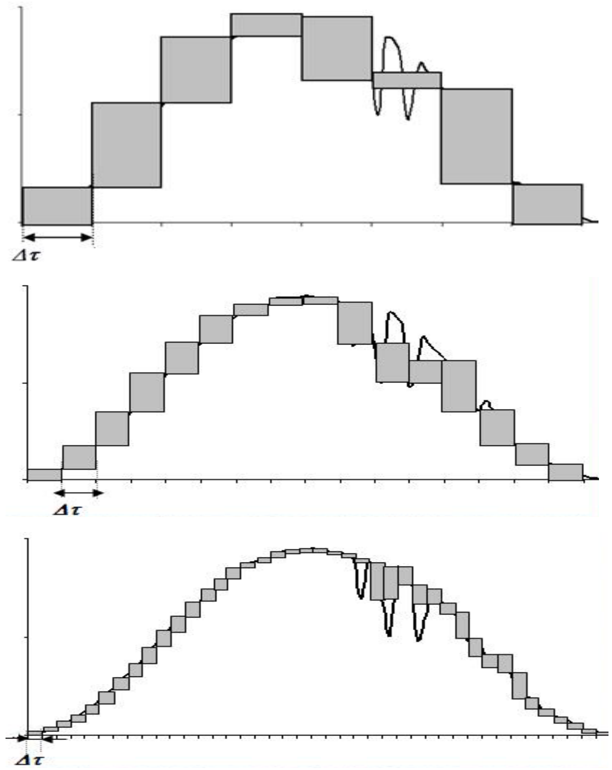


Fig. 1. An example of a curve covered by rectangles at different scales $\Delta\tau$

N denotes the signal length (number of samples of the considered signal), $f(t_n)$ is the value of the function representing the signal at the time t_n and $|f(t_n+\Delta\tau)-f(t_n)|$ is the function variation related to the interval $\Delta\tau$. Bouligand defined the fractal dimension D as follows (Bouligand, 1928):

$$D=2-\lambda(S) \qquad (2)$$

where $\lambda(S)$ is the similitude factor and it represents the infinitesimal order of $S(\Delta\tau)$. It is defined by:

$$\lambda(S)=\lim_{\Delta\tau\to 0}\left[\ln(S(\Delta\tau))/\ln(\Delta\tau)\right] \qquad (3)$$

ln is the logarithm Neperian.

Replacing $\lambda(S)$ by its value in (3) we obtain:

$$D=\lim_{\Delta\tau\to 0}\left[2-\left(\ln(S(\Delta\tau))/\ln(\Delta\tau)\right)\right] \qquad (4)$$

The properties of the logarithm permit us to put (4) under the following shape:

$$D=\lim_{\Delta\tau\to 0}\left[\ln\left(S(\Delta\tau)/\Delta\tau^2\right)/\ln(1/\Delta\tau)\right] \qquad (5)$$

The fractal dimension is then deduced from the following relation by using the least-squares estimation:

$$\ln\left(S(\Delta\tau)/\Delta\tau\right)=D\cdot\ln(1/\Delta\tau)+\text{constant},\quad \text{as } \Delta\tau\to 0 \qquad (6)$$

Thus, to determine the fractal dimension D which represents the slope of the straight line (6), it is necessary to use various time scales $\Delta\tau$ and to measure the corresponding area $S(\Delta\tau)$. We then obtain several points $(\Delta\tau_i, S(\Delta\tau_i))$ constituting the line (6).

A good estimation of the fractal dimension D requires a good fitting of the log-log plot defined by (6). Therefore, the number of points constituting the plot is important. This number is fixed by $\Delta\tau_{max}$ which is the maximum scale over which we attempt to fit the log-log plot.

To estimate the fractal dimension most of methods determine $\Delta\tau_{max}$ experimentally. This procedure requires much time and suffers from precision. Also, we developed an optimization technique to estimate $\Delta\tau_{max}$.

Figure 2, represents an example of log-log plot. Our optimization technique consists first in taking a $\Delta\tau_{max}$ minimal about 10, because the number of points constituting the plot should not be very small then, $\Delta\tau_{max}$ is incremented progressively as far as reaching $N/2$. We hence obtain several straight log-log lines which are fitted using the least squares estimation. The $\Delta\tau_{max}$ optimal is the one corresponding to the log-log straight line with the minimum quadratic error. This error is defined by the following formula:

$$E_{quad}=\sum_{i=1}^{n} d_i/n \qquad (7)$$

In this relation n denotes the number of points used for the straight log-log line fitting, d_i represents the distance between the points $(\ln(1/\Delta\tau), \ln(S(\Delta\tau)/\Delta\tau^2))$ and the fitted straight log-log line. This distance is calculated as showed in figure 3. In this figure $\ln(1/\Delta\tau)$ is denoted X and $\ln(S(\Delta\tau)/\Delta\tau^2)$denoted Y. The distance d_i is calculated from the difference between the co-ordinates Y of these points.

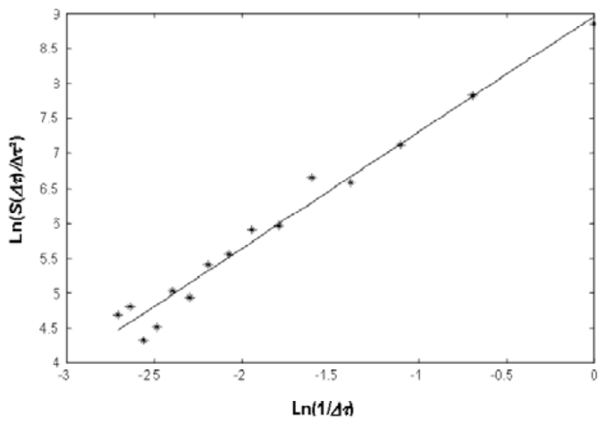


Fig. 2. An example of log-log plots fitted by the least squares estimation

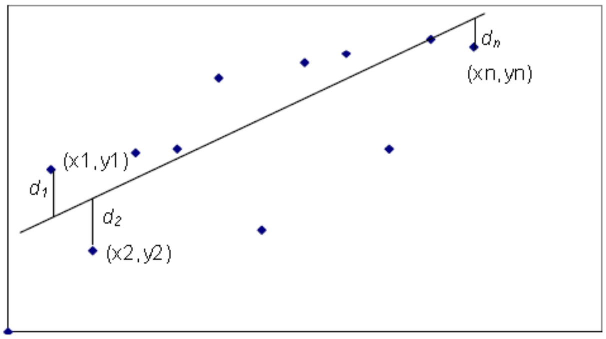


Fig. 3. Calculation of the distance d_i of real points constituting the log-log plots

3. QUANTIFICATION OF WIND SPEED FLUCTUATION

We propose here to measure the fractal dimension of wind speed series in other to quantify their fluctuations.

3.1 Data Bank Description

The data used in this study consist of hourly surface wind speeds (standard 10m level) after anemometer height adjustment recorded at the station of Quebec for the year of 2006. These data have been extracted from the National Climate Data Archive of Environment Canada (AHCCD, 2009).

Figure 4 shows the annual variability in wind speed frequency distribution. As can be seen from the figure the distribution can be approximated as many other wind speed distribution by the Weibull function.

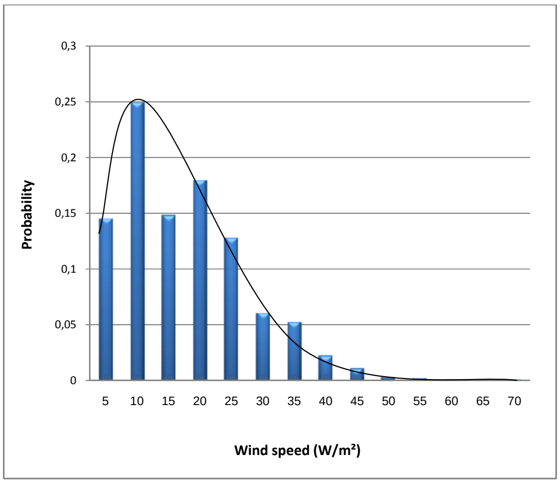


Fig. 4. Hourly Wind Speed Distribution

3.2 Fractal Treatment of Wind Speed

Using the Rectangular Covering Method we have estimated the fractal dimension of the wind speed series described above for each day of the year.

Figure 5 presents two examples of the log–log lines permitting the estimation of the fractal dimension of wind speed curves. This figure shows that the log–log points are grouped around the fitting line which demonstrates the self-affinity of the studied wind speed.

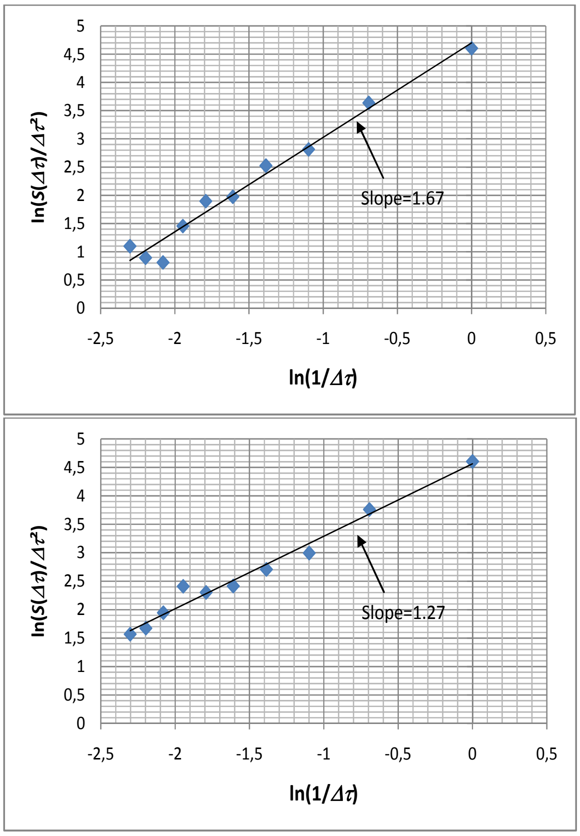


Fig. 5. Two examples of log-log plots fitted by the least squares estimation with their slopes which represent the estimated fractal dimension

As an illustration of the results, we give in the figure 6 the evolution of the estimated fractal dimension for the month of January.

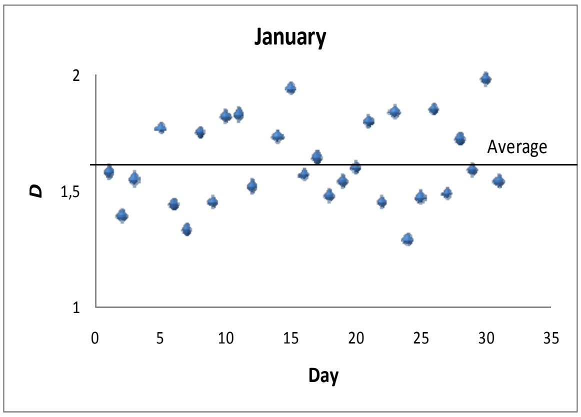
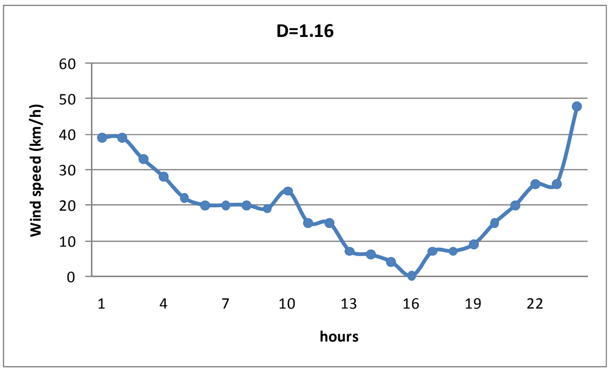
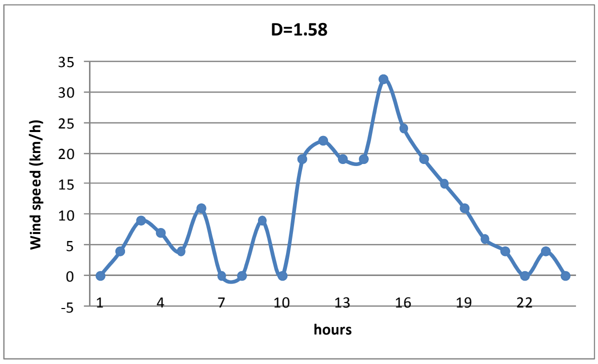


Fig. 6. Variation of the fractal dimension over the month of January

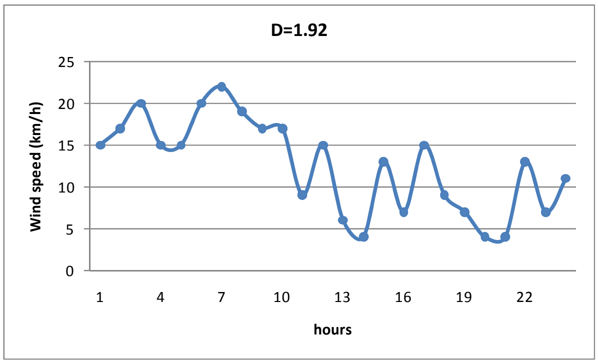
Figure 7 gives representative examples for three daily wind speed curves corresponding to different fractal dimensions. As can be observed there is good correspondences between the shapes of the signals and the corresponding fractal dimensions.



(a)



(b)



(c)

Fig. 7. Daily evolution of hourly wind speed with their fractal dimensions (a) February 04 1986 (b) May 05 1986 (c) August 10 1986

Table 1 presents the monthly average, maximum, and minimum fractal dimensions obtained from the slopes of the log–log lines.

Table 1 shows clearly that *D* fluctuates since the lowest value of this parameter is 1.16 and the highest one is 1.98. In order to quantify this fluctuation we calculated the annual average of *D* is equal to 1.64. To compare the degree of fluctuation of the hourly wind speed for the different months of the year we can refer to the average value of the fractal dimension for each month.

These values suggest that the hourly wind speed exhibit the similar fluctuations for all months.

Table 1. Monthly average, lowest, and highest fractal dimension for the year 1986

Month	Average value of *D*	Lowest value of *D*	Highest value of *D*
January	1.62	1.29	1.98
February	1.57	1.16	1.95
March	1.62	1.34	1.94
April	1.66	1.37	1.92
May	1.63	1.35	1.92
June	1.68	1.30	1.93
July	1.68	1.27	1.94
August	1.66	1.38	1.92
September	1.67	1.26	1.97
October	1.59	1.21	1.84
November	1.59	1.28	1.95
December	1.64	1.30	1.96

The other important parameter quantifying the wind speed fluctuation is the fractal dimension frequency distribution. This distribution describes the percentage of the fractal dimension between different levels. Hence, in the figure 8 we represent this distribution for the fractal dimension obtained.

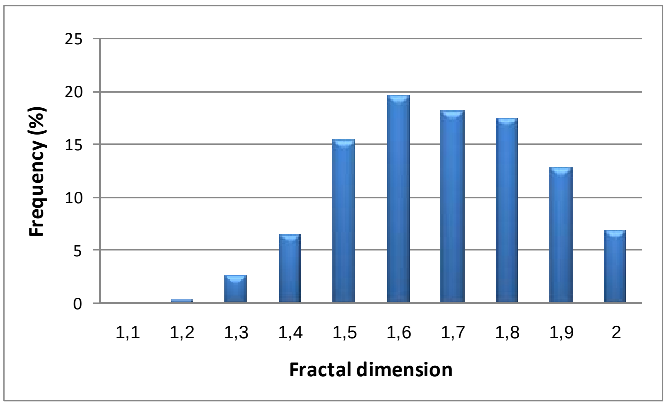


Fig. 8. Frequency distribution of the fractal

This figure shows that the fractal dimensions grouped between 1.4 and 1.9. This result confirms that the wind speed of the studied site is characterized by middle to high values of fractal dimensions which demonstrates that the hourly wind speed presents relatively high fluctuations.

4. CONCLUSION

We have shown in this work that the fractal analysis described well the wind speed fluctuations. The fractal dimensions estimated for hourly wind speeds in the site of Quebec using the Rectangular Covering Method we already developed suggested that this site is characterized as having winds with high degree of fluctuations in overall. This result plays an important role in the assessment of the efficiency of wind turbines installations in the studied site. Indeed, the wind speed fluctuations have a direct effect on the wind turbines response. Modelling of the wind fluctuations will then enable system operators to simulate the expected power fluctuations with a specified wind power development scenario. This is useful to calculate the need for reserves to balance the power fluctuations.

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