Study of Diffusion Coefficient in Sand piles

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Abstract: We simulated three sand pile models and studied the variation of their density of active sites and also their diffusion coefficient as a function of density of piles and, we observed that the diffusion coefficient has a meaningful relationship with active sites density in all three models. In the Manna's restricted model we saw phase transition in the active sites in the range 0.92-0.93 of piles density. While we have seen a non-monotonic behavior for its diffusion coefficient in which the maximum point is also in range 0.92-0.93 of piles density. In model A we observed these behaviors too, but the phase transition point is in the range 0.6-0.7 of piles density, and in model B basically there is no phase transition.

1. INTRODUCTION

Sand pile models are the important examples of self-organized criticality (SOC) [1, 2], The introduction of sand pile models by Bak, Tang and Wiesenfeld (BTW)[1] stimulated numerous theoretical[2,3] and numerical studies [4–7]. Large scale simulations of the BTW model [4] and some variants of it [8, 9] were performed. The BTW model and the Manna's model were concluded to belong to the same universality class [8]. In recent years considerable progress has been made in characterizing the critical properties of conserved stochastic sand piles, although no complete, reliable theory is yet at hand. In this paper we focus on diffusion coefficient, a parameter in sand piles that has received relatively little attention. Since the dynamics in these models involves hopping of particles between neighboring sites, one expects the particle diffusion constant D to follow a scaling behavior similar to that of the usual order parameter ρ, the active site density. Here D is defined as $((\Delta x)^2)=2Dt$, where Δx is the particle displacement.

This paper is organized as follows: In Sec. II, we define our three models. Sec III reports simulation results for diffusion coefficient, D and for density of active sites, ρ; for three models.

2. MODELS

We have simulated three models for sand piles and compared their results. The three models are as below:

2.1 Model A:

in this model in the initial state (before toppling) there are no limits for the number of piles in each site, and an active site is a site which has 2 or more piles. For toppling, an active site is chosen randomly and its piles fall in one of its two neighbor sites with equal probability unless this change makes the neighbor site active. For example if the chosen site has three(or even more) piles and both of its neighbors have one pile, the chosen site could not topple.

2.2 Model B:

this model has a big difference with Manna's Model and model A; activity of a site is defined with the difference between number of its piles and number of its neighbors' piles. If a site has two or more piles more than its right/left neighbor, that site is active relative to its right/left neighbor. A site could be active just relative to one of its neighbors, and there are no limits for the number of piles in each site. For toppling, an active site is chosen randomly and its piles (one by one) fall into the neighbor site(s) with equal probability, but to those neighbor site(s) which is allowed (according to the definition of model) to topple to them. If a site is not active it could not topple.

2.3 Manna's Restricted Model [10]:

in this model sites can not have more than two piles. Every site with two piles is an active site. Two piles of the chosen site could independently go to one of its neighbor sites which have less than two piles (with same probability).

3. SIMULATION

The simulated program is an object oriented code in java. There is three main objects: Pile, Site and Lattice. Each pile knows its location(the site that contains it) and has a counter to keep the number of its moves to calculate diffusion coefficient. Each site knows number of its piles and knows it's left and right neighbors. lattice knows total number of piles and number of sites. Each sandpile model is implemented as a function of Site class. we have implemented two more functions for testing the simulation: a function to make a custom initial state, and a function to select sites to topple manually to observe the functionality of simulated models. Because of the object-oriented code, it could be developed for other sandpile models and even for two dimensional lattices.

4. RESULTS

In our simulation, we took the density of piles as a variable, in each value of this density we let the piles to topple, and then we calculated the density of active sites, and also the average diffusion coefficient of the toppling piles. Then we fallowed the behaviour of the active site density, and also the diffusion coefficient as functions of pile density, we have done all these for all three models individually. In this section we have listed the above studies for three mentioned models.

4.1 Manna's restricted Model:

4.1.1 Density of active sites:

in our simulation, we calculated the density of active site in all densities of piles after the toppling of piles; we found that there is a phase transition between 0.92 and 0.93, a result which has been reported by other groups [1,10], figure(1)shows this result .

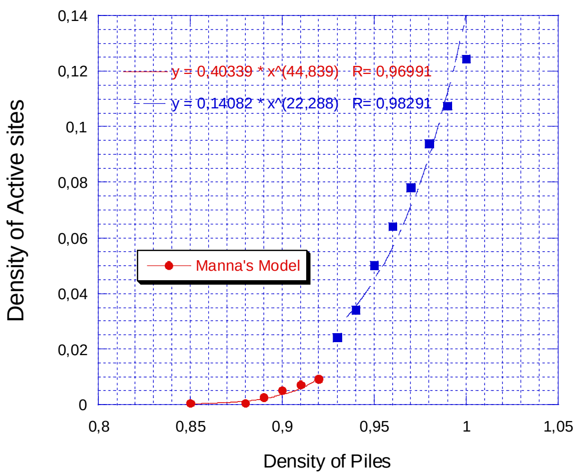

Figure 1: Density of active sites for different densities of piles for Manna's model. There is a phase transition in the range of 0.92-0.93.

4.1.2 Diffusion Coefficient:

We have also calculated the diffusion coefficient in all density of piles, we found that it is a non-monotonic function of density, and the function has a maximum between 0.92 and 0.93, figure (2) shows this result. This is in agreement with the results of Cunha *et,all*[11] who showed that the diffusion coefficient scales like active site density.

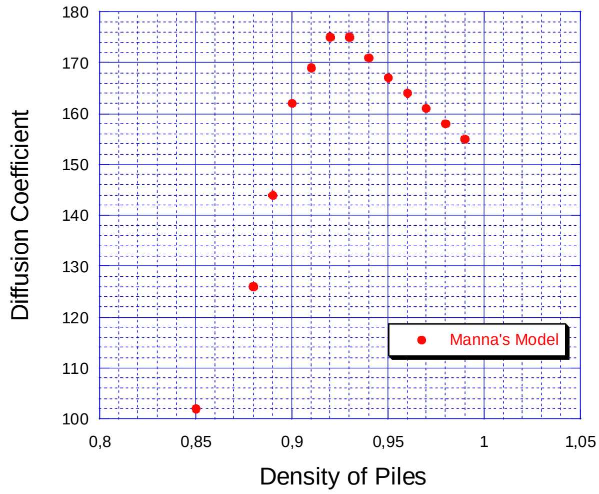

Figure 2: Diffusion coefficient for Manna's model, the non-monotonic function has a maximum in the range of 0.92-0.93.

4.2 Model A:

4.2.1 Density of active sites:

in this model, we have also a phase transition but in another point, the phase transition point shifts to the range of 0.6-0.7 for density of piles, figure (3) shows this result.

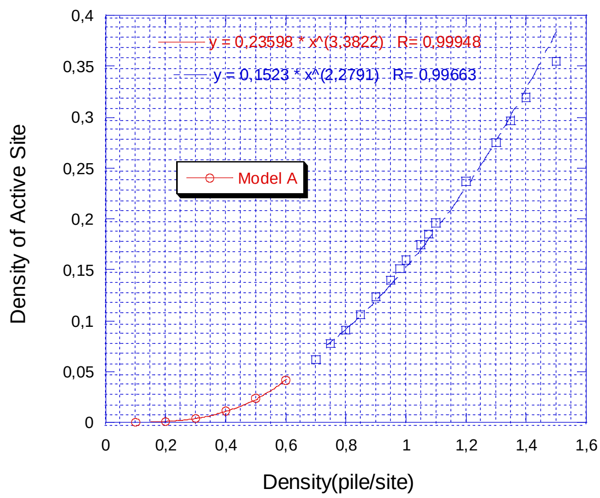

Figure 3: Density of active sites for different densities of piles for model A. There is a phase transition in the range of 0.6-0.7.

4.2.2 Diffusion Coefficient:

Also here we observed a non-monotonic behavior, but the maximum point is also shifted to the range of 0.6-0.7 for density of piles, in consistence with results obtained for density of active sites, figure (4) shows this result.

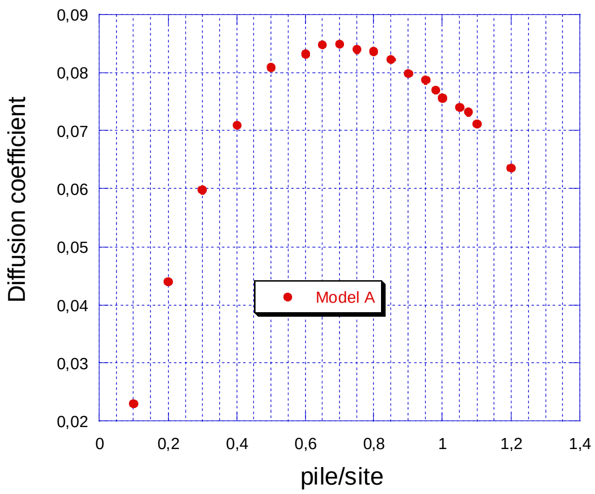

Figure 4: Diffusion coefficient for model A, the non-monotonic function has a maximum in the range of 0.6-0.75.

4.3 Model B

4.3.1 Density of active sites:

In this model the final value of the density of active sites will be zero in all densities of piles.

4.3.2 Diffusion Coefficient:

In this model the diffusion coefficient saturates to a final value, so we do not have any phase transition in this model, figure (5) shows this result.

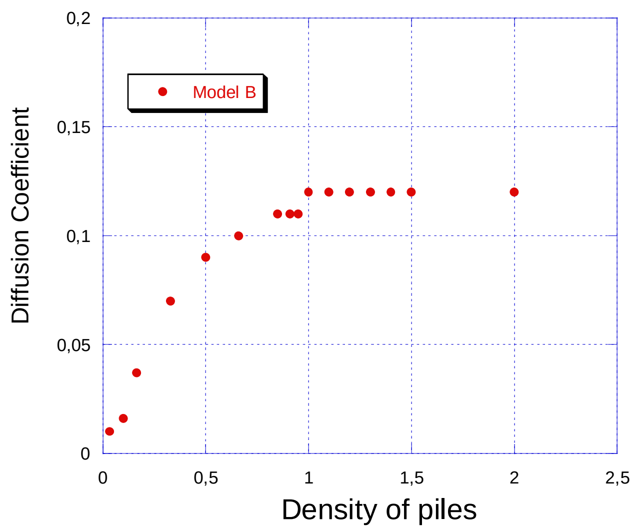

Figure 5: the diffusion coefficient in model B saturates to a final value and there is not a phase transition.

5. CONCLUSION

Our studies on Manna's model showed that the active site density has a phase transition in the range of 0.92-0.93 for pile density, a result which was also obtained by other groups [10, 11]. We have also found that the diffusion coefficient is a non-monotonic function of pile density which has a maximum point in the range of 0.92-0.93 for pile density. This result shows that the diffusion coefficient can be also regarded as an order parameter, which is in agreement with the results obtained by Cunha *et.all* [11] which showed that the diffusion coefficient scales like active site density.

We have also studied two other models, in model A we observed that we have a phase transition for active site density, but in another point, the transition point shifted to the range 0.6-0.7 of pile density. The interesting thing is that in this new point, the diffusion coefficient has a maximum again; it means that also in this model the diffusion coefficient can be regarded as an order parameter.

But in model B there is no phase transition, the final active site density is zero and the diffusion coefficient is a monotonic of pile density.

So we found that there are models other than the Manna's model which pose a phase transition but in another point, in this model we can regard the diffusion coefficient as an order parameter too. So the diffusion coefficient has a meaningful relationship with active sites density in all three models.

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