Quintom Model with the Double and Triple-Sine-Gordon Potentials

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Abstract: In this paper, we have been considered quintom model of dark energy by two non-interacting potentials as $V(\phi,\chi) = V_1(\phi) + V_2(\chi)$. The fields of ϕ and χ are introduced as quintessence and phantom fields respectively. We have also used double-sine-Gordon potential for quintessence field and triple-sine-Gordon potential for phantom field. The condition for the accelerated expansion of universe is obtained by equation of state (EoS). Thus we have showed that EoS parameter crossing of value -1. Finally we have plotted the corresponding graphs such as fields, the EoS parameter and potential in terms of time evolution. The investigation of bouncing universe condition will be interesting problem in future.

Keywords: Dark energy; Quintom model; Equation of state parameter; Sine-Gordon potential; quintessence field; phantom field.

1. INTRODUCTION

In recent decade, cosmological observations revealed a positive accelerating expansion of universe by SNe Ia (Riess et al. (1998) and Perlmutter et al. (1999)), WMAP (Bennett et al. (2003) and Spergel et al. (2003)) SDSS (Seljak et al. (2005)), Chandra X-ray observatory (Allen et al. (2005)) and so on. This cosmic acceleration represent a mysterious energy that called dark energy. We note that equation of state (EoS), i.e., pressure proportional to energy density can describe dark energy model during the evolution of the universe. In that case the EoS tend to value -1 in which one corresponds to astrophysical data. So that we know the problem of dark energy, accelerated expansion of the universe based on one or more dynamic scalar field described. For the EoS (ω) less than -1 the phantom dark energy (Caldwell (2002), Nojiri et al. (2003), Wei et al. (2004)) is observed, and for $-1 < \omega < -\frac{1}{3}$ the dark energy is described by quintessence (Ratra et al. (1988), Wetterich (1988), Caldwell et al. (1998)). We realize the properties of dark energy from recent observations with ω crossing -1 in the near past. Meanwhile for the phantom model of dark energy which has the opposite sign of the kinetic term compared with the quintessence in the Lagrangian. Neither the quintessence nor the phantom alone can fulfill the transition from $\omega > -1$ to $\omega < -1$ and vice versa. But one can show that considering the combination of quintessence and phantom in a joint model, the transition can be fulfilled. This model, called quintom (Feng et al. (2005), Guo et al. (2005), Setare et al. (2008)), can produce a better fit to the data than more familiar models with $\omega \geq -1$.

So far, a large class of scalar field dark energy models have been studied, including quintessence, phantom, quintom, tachyon (Sen (2002), Padmanabhan (2002)), K-essence (Armendariz-Picon et al (2000)) and so on. But we should note that the mainstream viewpoint regards the scalar field dark energy models as an effective description of an underlying theory of dark energy. In addition, other proposals on dark energy include interacting dark energy models (Zhang (2005)), braneworld models (Deffayet et al (2002)) and etc. Anyway we note that almost these models are determined from the phenomenology without theoretical root.

In order to see ω crossing to -1, we will need to introduce a potential as functional of scalar fields. Therefore we choose a combination of potentials as double-Sine-Gordon (DSG) and triple-Sine-Gordon (TSG), by motivated this potential describe the quintom model. The normal Sine-Gordon equation has been shown in different systems such as condensed matter, quantum optics, and particle physics. In this paper we use DSG potential for canonical field (quintessence) and TSG potential for non-canonical field (phantom). Also the quintom potential is taken by non-interacting potential as the following form,

$$V(\phi,\chi) = V(\phi) + V(\chi), \tag{1}$$

where $V(\phi)$ and $V(\chi)$ are functional of quintessence field and phantom field respectively. The mentioned quintom potential is written separably as DSG and TSG potentials by,

$$\begin{aligned} V(\phi) &= 1 + \alpha - \cos(\phi) - \alpha\cos(2\phi), \\ V(\chi) &= 1 + \beta - \cos(\chi) - \beta\cos(3\chi), \end{aligned} \tag{2}$$

where α and β are a constant. These potentials reduce to the ordinary Sine-Gordon potential in the limit $\alpha \to 0$ and $\beta \to 0$.
In the present paper, we suggest a quintom potential by adding DSG and TSG potentials. We show that this potential can describe quintom model of dark energy with ω crossing to -1.

2. THE QUINTOM MODEL OF DARK ENERGY

The quintom model of dark energy is a good scenario and one can explain the new astrophysical data for ω crossing to -1, i.e. transition from quintessence dominated universe to phantom dominated universe. Here we consider the spatially flat Friedman-Robertson-Walker (FRW) universe, where has following space-time metric,

$$ds^2 = dt^2 - a(t)^2(dr^2 + r^2 d\Omega), \tag{3}$$

where $a(t)$ is scale factor. The below action is introduced for quintom model that containing the normal scalar field ϕ and negative kinetic scalar field *chi*.

$$s = \int d^2x\, \sqrt{-g}\Big[-\frac{R}{4} + \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2} g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\phi,\chi)\Big], \tag{4}$$

we are using $4\pi G = 1$. The energy density and the pressure of the scalar fields can be written respectively by,

$$\rho = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\dot{\chi}^2 + V, \quad p = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\dot{\chi}^2 - V. \tag{5}$$

The equation of motion for two scalar fields in FRW model will have the following form,

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi = 0, \qquad \ddot{\chi} + 3H\dot{\chi} - V_\chi = 0\,. \tag{6}$$

The EoS can be written as,

$$\omega = \frac{\mathrm{p}}{\rho} = -1 - \frac{2}{3}\frac{\dot{\mathrm{H}}}{\mathrm{H}^2} = \frac{\dot{\phi}^2 - \dot{\chi}^2 - 2V}{\dot{\phi}^2 - \dot{\chi}^2 + 2V}, \tag{7}$$

in here we will see that for $\dot{\phi} > \dot{\chi}$, $\omega \geq -1$ and for $\dot{\phi} < \dot{\chi}$, we will have, $\omega < -1$.
By using Einstein equation ($R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 2T_{\mu\nu}$) and energy-momentum tensor ($T^\nu_\mu = diag\,(\rho, -p, -p, -p)$), we achieve the following Friedman' equations as,

$$H^2 = \frac{1}{3}\left(\dot{\phi}^2 - \dot{\chi}^2\right) + \frac{2}{3}V, \quad \dot{H} = -\left(\dot{\phi}^2 - \dot{\chi}^2\right). \tag{8}$$

Also we can obtain potential as,

$$V = \frac{1}{2}\left[3\mathrm{H}^2 + \dot{\mathrm{H}}\right]. \tag{9}$$

Now we can apply the above equations for DSG and TSG potentials.

3. DARK ENERGY WITH DSG AND TSG POTENTIALS

Here we consider quintom model of dark energy with DSG and TSG potentials. In order to describe dark energy

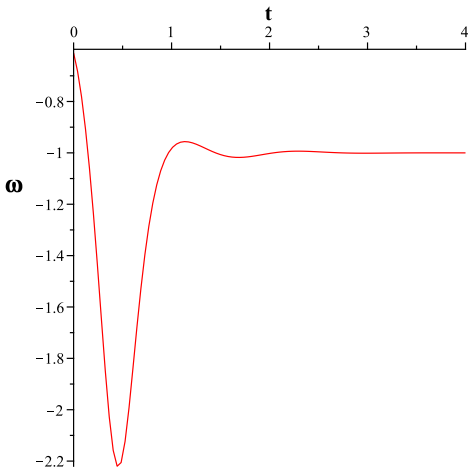

Fig. 1. Plot of the EoS with respect to time evolution.

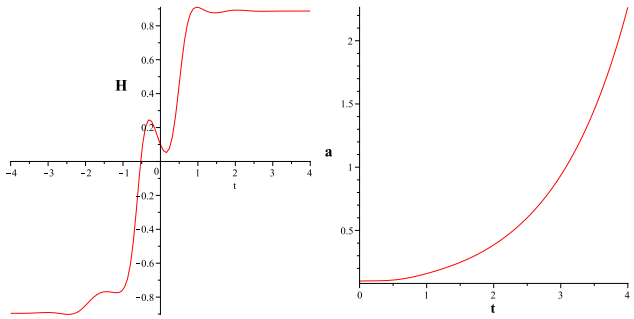

Fig. 2. Plots of the Hubble parameter and scale factor with respect to time evolution.

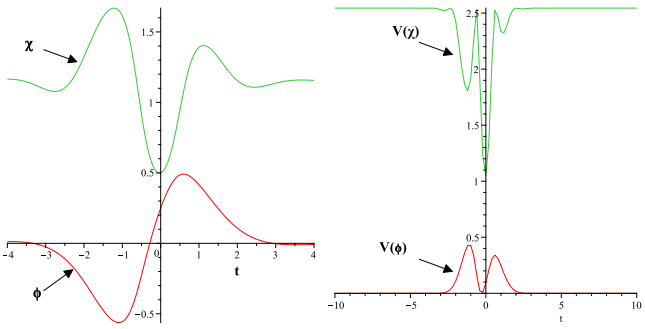

Fig. 3. Plots of the fields ϕ and χ, and corresponding potentials with respect to time evolution.

by quintom model, we should see to cross ω to -1. By inserting Eq. (2) into Eqs. (6) we get,

$$\begin{aligned} &\ddot{\phi} + 3H\dot{\phi} + \sin(\phi) + 2\alpha\sin(2\phi) = 0, \\ &\ddot{\chi} + 3H\dot{\chi} - \sin(\chi) - 3\beta\sin(3\chi) = 0\,. \end{aligned} \tag{10}$$

Now by numerical solution Eqs. (8) and (10) and choosing $\phi(0) = 0.25$, $\chi(0) = 0.5$, $\phi'(0) = 0.75$ and $\chi(0) = 0.1$, we can draw the EoS, fields, potentials and Hubble parameter with respect to time evolution. In Fig. 1, we can see various of the EoS in terms of time evolution for some choice of parameters and one shows that ω crosses of -1.

In Fig. 2 we plot $H(t)$ and $a(t)$ for some choice of parameters. Various of the Hubble parameter confirms $\dot{H} > 0$ in which one satisfies condition $\dot{\phi}^2 - \dot{\chi}^2 < 0$, i.e., we

will have $\omega < -1$. Also Various of the scale factor shows us accelerated expanding of universe. The Fig. 3 shows $\phi(t)$, $\chi(t)$ and $V(t)$.

4. CONCLUSION

In this study, the accelerated expansion of the universe took the help of real scalar fields by procedure of quintom model. We have taken DSG and TSG potentials in this model and we have showed the accelerated expansion of the universe by the parameter equation of state and we saw crossing the line separator phantom. Therefore we can say that DSG and TSG potentials are a good answer for this model.

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