Adaptive Active Sliding Mode Control for Two-Degree of Freedom Polar Robot

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Abstract: This paper presents developing an adaptive active sliding mode controller for a robotic manipulator in the presence of external disturbances. Primarily, two sliding surfaces are employed. Then, an active sliding mode control law is derived. Lyapunov stability theorem has been used to ensure that the proposed controller guarantees the system stability even in the presence of external disturbances. An adaptation law is derived for the control parameters based on a Lyapunov function candidate. By using this adaptation law there is no need to know the bounds of external disturbances. For the problem of determining appropriate values of the design parameters Particle Swarm Optimization (PSO) algorithm has been used which is the paradigms of swarm intelligence. Simulation studies have been carried out to verify effectiveness of the proposed controller.

Keywords**:** Adaptive Active Sliding Mode Control, Robotic Manipulators, Lyapunov Stability Theorem, Adaptation Law, Particle Swarm Optimization (PSO).

1. INTRODUCTION

Robot manipulators are typical examples of trajectory controllable mechanical systems. However, their highly nonlinear dynamics present a challenging control problem. Moreover there always exist uncertainty and external disturbances which cause undesired performance. In recent years, as the need for achieving greater accuracy in tracking problems there has been a surge of interest in using robust nonlinear control techniques such as sliding mode control(Huh *et al.,* 2007), adaptive control(Zeinali *et al.,* 2010), fractional-order control (Delavari *et al.*, 2010) intelligent approaches (Patiño, *et al.*, 2002) and etc. The aim of this paper is developing an adaptive active sliding mode control technique which provides high performance tracking and good robustness in spite of external disturbances and parameter variations.

Active sliding mode control which is composed of two control techniques; the active control and the sliding mode control is one of the well-known control techniques that are used in control problems. Recently, this control technique has been used for synchronization of fractional-order chaotic systems (Tavazoei, *et al.,* 2008). We have derived an adaptation scheme for the active sliding mode controller parameters in spite of uncertainties and external disturbances. By using this adaptation law there is no need for the bounds of disturbances (Roopaei, *et al.*, 2010). So, the controller becomes more robust and efficient. For the problem of determining the design parameters Particle Swarm Optimization (PSO) algorithm has been used.

PSO was introduced by Kennedy and Eberhart in 1995 (Kennedy *et al*., 1995; Eberhart *et al.,* 1995). The PSO uses a simple mechanism that mimics swarm behavior in birds flocking and fish schooling to guide the particles to search for globally optimal solutions. As PSO is easy to implement, it has rapidly progressed in recent years and with many successful applications seen in solving real-world optimization problems (Lazinica *et al.,* 2009*;* Faieghi *et al.,* 2010).

This paper is organized as follows: in section 2 the mathematical model of robot manipulator is given. Section 3 deals with developing the active sliding mode control. Section 4 presents stability analysis of the proposed controller. In section 5 an adaptation law is determined and in section 6 we have used PSO to find appropriate values of design parameters. Simulation results are illustrated in section 7 and finally, we conclude the paper in section 8.

2. MANIPULATOR DYNAMICS

A manipulator is defined as an open kinematics, chain of rigid links. Each degree of freedom of manipulator is powered by independent torques. In the absence of friction or other disturbances, dynamic of an *n*-link rigid robotic manipulator system can be described by the following second order nonlinear vector differential equation:

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)=\tau \quad (1)$$

where $q,\dot{q},\ddot{q}\in R^n$, q joint variable n-vector and τ is n-vector of generalized forces. $M(q)\in R^{n\times n}$ is a symmetric and positive definite inertia matrix, $C(q,\dot{q})\dot{q}$ is coriolis/centripetal vector, and $G(q)$ is the gravity vector. In general, a robot manipulator always presents uncertainties such as frictions and disturbances. The controller has a duty to overcome these problems (Slotine *et al.,* 1991).

2.1 Two-degree of freedom polar robot manipulator

As shown in Fig.1 a two-degree of freedom polar robot manipulator has one rotational and sliding joint in the (x,y) plane. Neglecting the gravity force and normalizing the mass and length of the arm, a mathematical model of two-degree of freedom polar robot can be expressed as follows:

$$\begin{aligned}
\dot{x}_1 &= x_2\\
\dot{x}_2 &= \left\{[\mu x_1+M(x_1+a)x_4^2]+u_1+d_1\right\}\Big/(\mu+m)\\
\dot{x}_3 &= x_4\\
\dot{x}_4 &= \left\{-2\begin{bmatrix}M(x_1+a)x_2x_4\\+\mu x_1+u_2+d_2\end{bmatrix}\right\}\Bigg/\begin{bmatrix}J_1+J_2+\mu x_1^2\\+M(x_1+a)^2\end{bmatrix}
\end{aligned} \quad (2)$$

where μ is the mass of motional link, M is the payload, J_1 and J_2 are moments of inertia of the motional link with respect to the vertical axis through c and o, respectively. $d_i(t)$ is an unknown but bounded external disturbance, i.e.:

$$|d_i(t)|\le D_i, i=1,2 \quad (3)$$

3. ACTIVE SLIDING MODE CONTROL

In this section we will derive an active sliding control law for control of robot manipulator described by (2). Assume that $x_{d1},x_{d2},x_{d3},x_{d4}$ are desired trajectories of states of (2). The tracking error is defined as follows:

$$\begin{aligned}
e_1 &= x_1-x_{d1}\\
e_2 &= x_2-x_{d2}\\
e_3 &= x_3-x_{d3}\\
e_4 &= x_4-x_{d4}
\end{aligned} \quad (4)$$

The tracking error dynamics is driven by substituting (2) in (4):

$$\begin{aligned}
\dot{e}_1 &= e_2\\
\dot{e}_2 &= \dot{x}_{d2}+\left\{[\mu x_1+M(x_1+a)x_4^2]+u_1+d_1\right\}\Big/(\mu+m)\\
\dot{e}_3 &= e_4\\
\dot{e}_4 &= \dot{x}_{d4}+\left\{-2\begin{bmatrix}M(x_1+a)x_2x_4\\+\mu x_1+u_2+d_2\end{bmatrix}\right\}\Bigg/\begin{bmatrix}J_1+J_2+\mu x_1^2\\+M(x_1+a)^2\end{bmatrix}
\end{aligned} \quad (5)$$

Now the tracking problem is changed into stabilization problem. The control problem is determining u_1 and u_2 in such a way that tracking errors are set into zero. First we choose two sliding surfaces as follows:

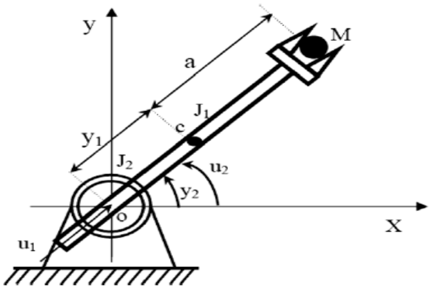

Fig.1 A two-degree of freedom polar robot manipulator

$$s_1=e_2+\lambda_1 e_1 \quad (6)$$

and

$$s_2=e_4+\lambda_2 e_3 \quad (7)$$

where λ_1 and λ_2 are two positive constants. The equivalent control law is proposed as follows:

$$\begin{aligned}
u_{eq,1} &= (\mu+m)\left\{-[\mu x_1+M(x_1+a)x_4^2]-\dot{x}_{d2}-\lambda_1\dot{e}_1\right\}\\
u_{eq,2} &= \begin{bmatrix}J_1+J_2+\mu x_1^2\\+M(x_1+a)^2\end{bmatrix}\left\{2\begin{bmatrix}M(x_1+a)x_2x_4\\+\mu x_1+u_2+d_2\end{bmatrix}-\dot{x}_{d4}-\lambda_2\dot{e}_3\right\}
\end{aligned} \quad (8)$$

The next step is to design the reaching mode control scheme, which drives the system trajectories onto the sliding surface. The active sliding mode control law is proposed as follows:

$$\begin{aligned}
u_{ASMC,1} &= k_1\,\mathrm{sgn}(s_1)+r_1 s_1\\
u_{ASMC,2} &= k_2\,\mathrm{sgn}(s_2)+r_2 s_2
\end{aligned} \quad (9)$$

where design parameters; k_1,k_2,r_1,r_2 are constants. The overall control law becomes:

$$
\begin{aligned}
u_1 &= u_{eq,1} + u_{ASMC,1} \\
u_2 &= u_{eq,2} + u_{ASMC,2}
\end{aligned} \tag{10}
$$

Next section devotes how to choose the design parameters.

4. STABILITY ANALYSIS

In this section we will use Lyapunov stability theorem to analyze the stability of the system (5) in the presence of control input (10). The error dynamics (5) is reduces to (11) by applying the control input (5):

$$
\begin{aligned}
\dot{e}_1 &= e_2 \\
\dot{e}_2 &= d_1 + k_1 \operatorname{sgn}(s_1) + r_1 s_1 - \lambda_1 e_1 \\
\dot{e}_3 &= e_4 \\
\dot{e}_4 &= d_2 + k_2 \operatorname{sgn}(s_2) + r_2 s_2 - \lambda_2 e_2
\end{aligned} \tag{11}
$$

Let us consider the following Lyapunov function candidate:

$$V = (s_1^2 + s_2^2)/2 \tag{12}$$

Taking derivative of both sides of (12) with respect to time, one has:

$$\dot{V} = s_1 . \dot{s}_1 + s_2 . \dot{s}_2 \tag{13}$$

From (6) and (7) we know that:

$$\dot{s}_1 = \dot{e}_2 + \lambda_1 \dot{e}_1 \tag{14}$$

and

$$\dot{s}_2 = \dot{e}_4 + \lambda_2 \dot{e}_3 \tag{15}$$

Substituting (14) and (15) into (13) and from (11), one has:

$$
\begin{aligned}
\dot{V} &= s_1 . \dot{s}_1 + s_2 . \dot{s}_2 \\
&= s_1(\dot{e}_2 + \lambda_1 \dot{e}_1) + s_2(\dot{e}_4 + \lambda_2 \dot{e}_3) \\
&= s_1(d_1 + k_1 \operatorname{sgn}(s_1) + r_1 s_1) + s_2(d_2 + k_2 \operatorname{sgn}(s_2) + r_2 s_2) \\
&= + r_1 s_1^2 + r_2 s_2^2 + s_1 d_1 + k_1 | s_1 | + s_2 d_2 + k_2 | s_2 |
\end{aligned} \tag{16}
$$

We should determine the values of design parameters to have $\dot{V} \le 0$ which guarantees asymptotic stability. From (16) we can understand that for any negative value of r_1 and r_2 the first two terms of (16) are non-positive. From the previous section we know that d_1 and d_2 are bounded with D_1 and D_2, respectively. So choosing $-k_1 \ge D_1$ and $-k_2 \ge D_2$ yields $s_1 d_1 + k_1 | s_1 | \le 0$ and $s_2 d_2 + k_2 | s_2 | \le 0$. So, we can choose design parameters in such a way that make the system stable.

5. ADAPTATION LAW SYNTHESIS

In the previous section it was assumed that we know the disturbances bounds. In this section we will derive an adaptive scheme for the design parameters which guarantees system stability without any knowledge of external disturbance.

Consider the following Lyapunov function candidate:

$$V_1 = (s_1^2 + \gamma_1^{-1} \tilde{k}_1^2 + \gamma_2^{-1} \tilde{r}_1^2)/2 \tag{17}$$

where $\tilde{k}_1 = k_1 - \hat{k}_1$, $\tilde{r}_1 = r_1 - \hat{r}_1$, taking derivative of both sides of (17) with respect to time yields:

$$\dot{V}_1 = s_1 . \dot{s}_1 + \gamma_1^{-1} \tilde{k}_1 . (-k_1) + \gamma_2^{-1} \tilde{r}_1 . (-r_1) \tag{18}$$

Using the control input (10) and substituting (16) into the (18), one has:

$$
\begin{aligned}
\dot{V}_1 &= s_1(d_1 + k_1 \operatorname{sgn}(s_1) + r_1 s_1) \\
&\quad + \gamma_1^{-1} \tilde{k}_1 . (-\dot{k}_1) + \gamma_2^{-1} \tilde{r}_1 . (-\dot{r}_1) \\
&\le D_1 | s_1 | + k_1 | s_1 | + r_1 s_1^2 \\
&\quad + \gamma_1^{-1} \tilde{k}_1 . (-\dot{k}_1) + \gamma_2^{-1} \tilde{r}_1 . (-\dot{r}_1) \\
&\le D_1 | s_1 | + k_1 | s_1 | + \hat{k}_1 | s_1 | - \hat{k}_1 | s_1 | \\
&\quad \hat{r}_1 s_1^2 - \hat{r}_1 s_1^2 + r_1 s_1^2 + \gamma_1^{-1} \tilde{k}_1 . (-\dot{k}_1) + \gamma_2^{-1} \tilde{r}_1 . (-\dot{r}_1)
\end{aligned} \tag{19}
$$

We can rewrite (19) as follows:

$$
\begin{aligned}
\dot{V}_1 &\le \left(D_1 + \hat{k}_1\right) | s_1 | + \hat{r}_1 s_1^2 + \left(k_1 - \hat{k}_1\right) | s_1 | \\
&\quad + \left(r_1 - \hat{r}_1\right) s_1^2 + \gamma_1^{-1} \tilde{k}_1 . (-\dot{k}_1) + \gamma_2^{-1} \tilde{r}_1 . (-\dot{r}_1)
\end{aligned} \tag{20}
$$

It is clear that $\hat{k}_1$ and $\hat{r}_1$ can be chosen in such a way that $\left(D_1 + \hat{k}_1\right) | s_1 | + \hat{r}_1 s_1^2 \le 0$, so the adaptation law can be derived directly from (20) as follows:

$$\left(k_1 - \hat{k}_1\right) | s_1 | + \gamma_1^{-1} \tilde{k}_1 . (-\dot{k}_1) = 0 \rightarrow \dot{k}_1 = \gamma_1^{-1} | s_1 | \tag{21}$$

$$\left(r_1 - \hat{r}_1\right) s_1^2 + \gamma_2^{-1} \tilde{r}_1 . (-\dot{r}_1) = 0 \rightarrow \dot{r}_1 = \gamma_2^{-1} s_1^2 \tag{22}$$

Similar to k_1 and r_1, by using the following Lyapunov function candidate:

$$V_2 = (s_2^2 + \gamma_3^{-1} \tilde{k}_2 + \gamma_4^{-1} \tilde{r}_2)/2 \tag{25}$$

adaptation law for k_2 and r_2 is determined as follows:

$$\left(k_2 - \hat{k}_2\right) | s_2 | + \gamma_3^{-1} \tilde{k}_2 . (-\dot{k}_2) = 0 \rightarrow \dot{k}_2 = \gamma_3^{-1} | s_2 | \tag{23}$$

$$\left(r_2-\hat{r}_2\right)s_2^2+\gamma_4^{-1}\tilde{r}_2.(-\dot{r}_2)=0\rightarrow\dot{r}_2=\gamma_4^{-1}s_2^2 \tag{24}$$

Block diagram of the control system is depicted in Fig.2.

6. PARTICLE SWARM OPTIMIZATION

In PSO, a swarm of particles are represented as potential solutions, and each particle i is associated with two vectors, i.e., the velocity vector $V_i=[v_i^1,v_i^2,...,v_i^D]$ and the position vector $X_i=[x_i^1,x_i^2,...,x_i^D]$ where D stands for the dimensions of the solution space. The velocity and the position of each particle are initialized by random vectors within the corresponding ranges. During the evolutionary process, the velocity and position of particle i on dimension d are updated as

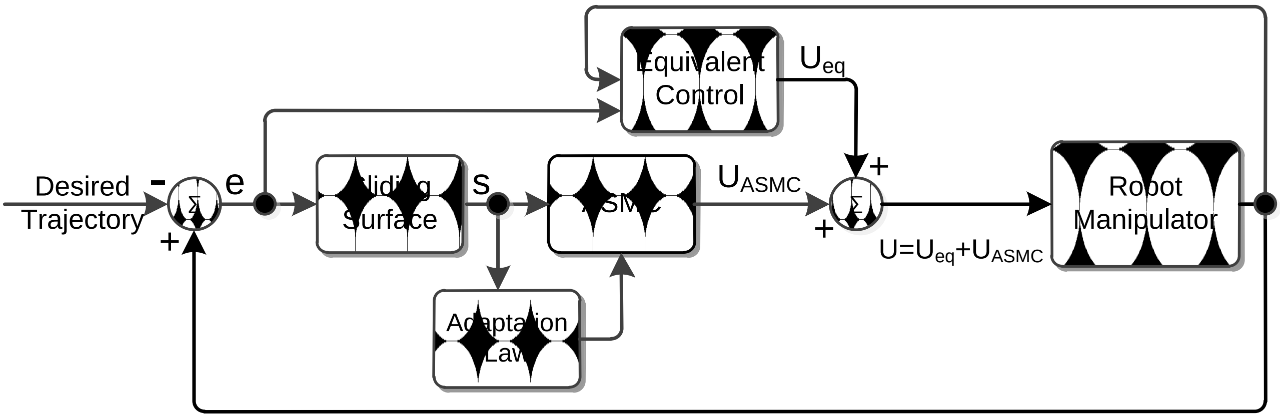

Fig.2 Block diagram of the proposed controller

$$\begin{aligned}v_i^d&=\omega v_i^d+c_1rand_1^d(pBest_i^d-x_i^d)\\&\quad+c_2rand_2^d(nBest^d-x_i^d)\end{aligned} \tag{26}$$

$$x_i^d=x_i^d+v_i^d \tag{27}$$

Where ω is inertia weight, c_1 and c_2 are acceleration coefficients, $rand_1^d$ and $rand_2^d$ are random numbers within $[0,1]$ interval for the d th dimension. In ‘‘Equation (9)’’ $pBest_i$ is the position with the best fitness found so far the i th particle, and $nBest$ is the best fitness position in neighborhood.

Here, PSO is used to search the parameter space to find appropriate values of design parameters by minimizing the cost function. The cost function that is used here is well-known cost function i.e. Root Mean Square Error (RMSE) which is defined as follows:

$$\sum_j E_j^1+E_j^2+E_j^3+E_j^4 \tag{28}$$

where E^i is the RMS value of e_i.

7. SIMULATION STUDY

To verify the effectiveness of the controller, simulation studies have been carried out by MATLAB/SIMULNK software. The parameters of the robot manipulator are set as: $M=1.5\,kg,\ m=0.5kg,\ \mu=1kg,\ J_1=J_2=1kg.m^2$ and $a=1\,m$, External disturbances are modeled as $d_1(t)=0.3\cos(4\pi t)$ and $d_2(t)=0.5\cos(4\pi t)$. Also we applied 20% parameter variation. Initial conditions are set as follows:

$$[x_1(0),x_2(0),x_3(0),x_4(0)]^T=[-0.2,-0.25,0.36,0.98]^T$$

Desired trajectories are:

$$\begin{aligned}x_{d1}&=0.5\cos(\pi t/7)\ m\\x_{d3}&=\pi\cos(\pi t/7)\ rad\end{aligned} \tag{29}$$

We have used PSO to find appropriate values of design parameters. Parameters of PSO are set as follows:

Population Size = 50, Number of Iterations = 50, Inertia Weight = 0.9 and $c_1=c_2=2.05$.

Parameters of the controller have found as follows:

$$\begin{aligned}&\lambda_1=1.5\ \ \lambda_2=1.0284\ \ \gamma_1=9.2053\\&\gamma_2=1.5\ \ \gamma_3=1.1748\ \ \gamma_4=20\end{aligned}$$

The simulation results are depicted in Figs (3-6) where Fig.3 shows tracking the desired trajectories defined by (29), Fig.4 shows the control input, Fig.5 is sliding surfaces trajectories during the tracking and Fig.6 shows the variation of design parameters according the adaptation law.

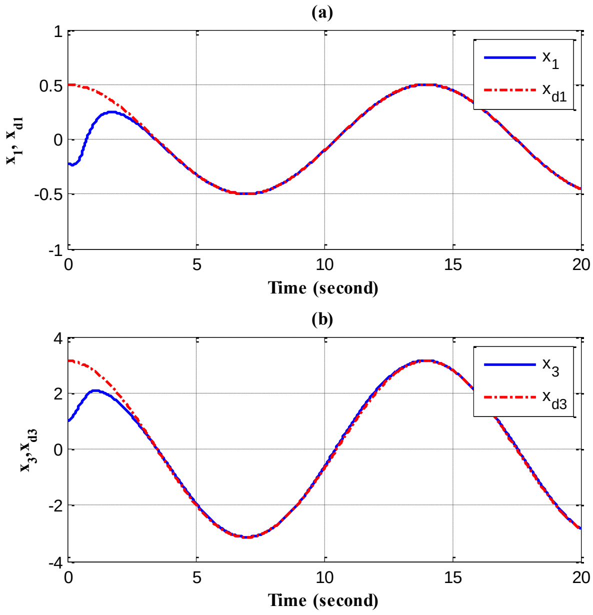

Fig.3 Tracking the desired trajectories (a) x_1 (b) x_3

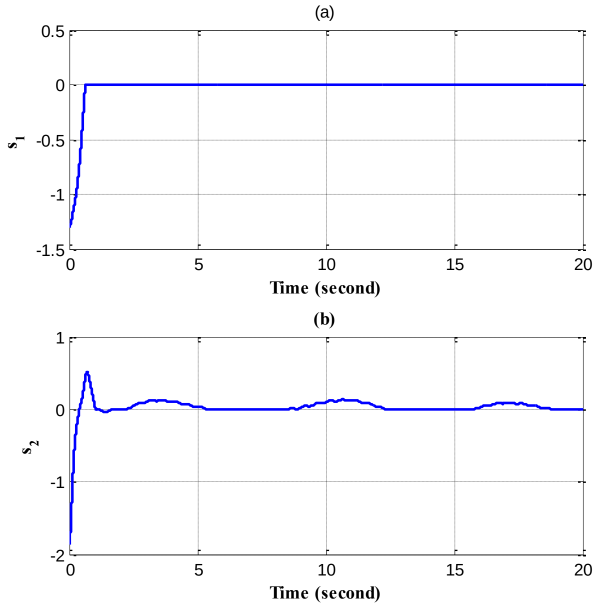

Fig.5 Sliding surfaces trajectory (a)s_1 (b)s_2

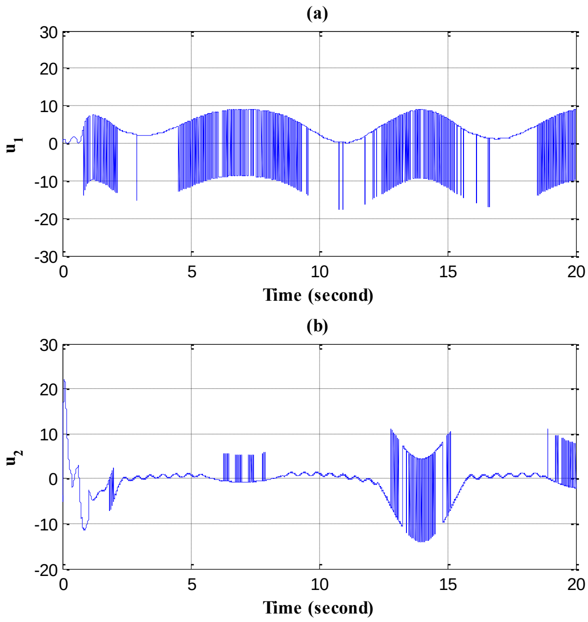

Fig.4 Control input during the tracking (a) u_1 (b) u_2

As shown in simulation results, the proposed controller can track the desired trajectories in the presence of external disturbances and parameter variation. By using the adaptation law there is no need to know the bounds of external disturbances. Fig. 6 shows how the parameters are adapted to make the system stable in the tracking procedure.

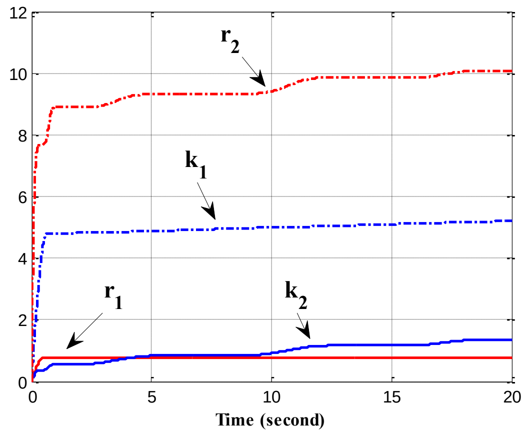

Fig.6 Time response of adaptation parameters

8. CONCLUSION

In this paper a novel intelligent adaptive active sliding mode controller for a robot manipulator is presented. The proposed controller operates well and a robust trajectory control in the presence of external disturbances and system uncertainties is achieved. The closed loop system stability is proved based on Lyapunov stability analysis. PSO is used to find appropriate values of the controller parameters. The major contribution of this paper is developing a novel adaptation law for the switching control action which prompts the controller to stabilize the system without knowledge of the bounds of disturbances and uncertainties, unlike many well-known methods of sliding mode control. Simulation results have been carried out to verify the significance of the controller. Numerical simulations show that starting from any initial conditions of the system states; the system outputs will asymptotically track the desired trajectories with a fast and robust response.

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