GEOMETRICALLY NONLINEAR FREE TRANSVERSE VIBRATION OF 2-DOF SYSTEMS INVOLVING THE COUPLING BETWEEN AXIAL AND TRANSVERSE DISPLACEMENTS

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Abstract Il is well known that large transverse vibration amplitudes of beams induces a geometrical non-linear behavior due to the axial displacements and strains, which are usually neglected in linear theory, but have a significant effect at large vibration amplitudes as shown both theoretically and experimentally in (R.Benamar, M.M.Bennouna, and R.G.White 1991). The purpose of the present work is to model such a behavior using a 2 dof system made of two masses and four spiral springs, similar to those recently used in (A. Eddanguir, Z. Beidouri, and R. Benamar 2009), in addition to three axial springs representing the non-linear axial displacements, induced by large transverse vibration amplitudes. Details of the theory are given, leading to definition of the mass, rigidity and non-linear rigidity tensors, usually encountered in previous works, dealing with non-linear vibration of structures of various types and geometry. Some numerical results are given, showing the amplitude dependence of the frequencies with the amplitude of vibration. The analogy between continuous beams and the discrete model, leading to the expressions for the equivalent spiral and axial stiffness will be presented later.

Keywords: Nonlinear transverse vibration, Two-degrees-of-freedom, Discrete system, Hamilton's Principle, Spectral analysis, Displacement Basis, Modal Basis, Nonlinear coupled longitudinal and axial vibrations.

1. INTRODUCTION

The purpose of the present work is to model a geometrical non-linear behavior of a system constituted by 2 masses interconnected by four linear spiral torsional springs similar to those used in (A. Eddanguir, Z. Beidouri, and R. Benamar 2009), in addition to three linear longitudinal spring (see figure 1).

Such a simplifed model may represent in a discretised form the non-linear flexural vibration of beams. The system is supposed to be conservative (no damping is involved). The proposed definition of the non-linear normal modes is based on the observation that the normal solutions of the linear system are all periodic of the same period, and that the ratio of the displacement of any mass to that of any other is identically equal to a constant for all time (R. M. Rosenberg, 1962).

The aim of this work is to take into account the nonlinear axial vibrations induced by the transverse vibration examined previously by many researchers (A. Eddanguir, Z. Beidouri, and R. Benamar 2009; Z. Beidouri, A. Eddanguir, and R. Benamar 2008, M. EL Kadiri, R. Benamar and R. G. White, 2002 a,b) , in order to determine the non-linear mode shapes of the structure shown in figure 1. Details of the theory are given, leading to definition of the mass, rigidity and non-linear rigidity tensors, usually encountered in previous works, dealing with non-linear vibration of structures of various types and geometry. In spite of the fact that the longitudinal springs are linear, their stretching, due to high vibration amplitudes, depend non-linearly on transverse displacements, as shown below. This is what is usually designed as geometrical non-linearity.

2. GENERAL FORMULATION

2.1 Presentation and nomenclature

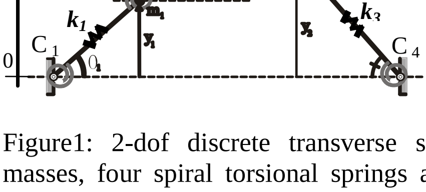

Figure1: 2-dof discrete transverse system made of two masses, four spiral torsional springs and three longitudinal springs.

Figure1 shows a 2-dof system made of 2 masses m_1, m_2 and four spiral torsional springs. C_r are the linear rigidities of the r^{th} spiral torsional spring, r =1 to 4. The momentum M in the spiral torsional spring is given by: $M = -C\Delta\theta$. The springs are attached by three bars considered as longitudinal spring having length l_i, i =1 to 3. These springs are supposed to be massless and have stiffnesses k_i i = 1 to 3.
The transverse displacements of the masses m_1 and m_2 from the unstretched positions of the springs are denoted as y_1 and y_2. The vector $\{y\}$ defined by $\{y\}^T = [y_1\ y_2]$ can be written as:

$$\{y\} = y_1\vec{u}_1 + y_2\vec{u}_2 = \bar{y}_1\vec{\phi}_1 + \bar{y}_2\vec{\phi}_2 \qquad (1)$$

where $\{\vec{u}_1, \vec{u}_2\}$ is the Displacement Basis (DB) defined by the vectors $\{u_1\}^T = [1\ 0]$, $\{u_2\}^T = [0\ 1]$. $\{\vec{\phi}_1, \vec{\phi}_2\}$ is the Modal Basis (MB), in which $\vec{\phi}_r$ represents the r^{th} linear mode shape of the 2-dof system. The components of the displacements of the masses m_1 and m_2 in DB and MB are (y_1, y_2) and $(\bar{y}_1, \bar{y}_2)$ respectively. In what follows, the components of $\vec{\phi}_r$ in DB are denoted as ϕ_{r1}, ϕ_{r2} . The transition matrix from DB to MB is the matrix of column vectors $\vec{\phi}_r$.

2.2 Expressions for the masse tensor, linear and non linear rigidity tensor in DB

Assume a harmonic motion defined by:

$$\begin{aligned} y_1 &= A_1\cos(\omega t) = (a_1\phi_{11} + a_2\phi_{12})\cos(\omega t) \\ y_2 &= A_2\cos(\omega t) = (a_1\phi_{21} + a_2\phi_{22})\cos(\omega t) \end{aligned} \qquad (2)$$

where the A_i's are the modulus of displacement y_i expressed in DB (or the contributions of normalised vectors of DB), a_i is the modulus of displacement y_i expressed in MB (or contributions of normalised vectors of MB).

The kinetic, linear and non-linear potential energies of the 2-dof system considered can be written as:

$$T = \frac{1}{2}\dot{y}_i\dot{y}_j m_{ij} = \frac{1}{2}a_i a_j\omega^2\bar{m}_{ij}\sin^2(\omega t) \quad i, j = 1,2 \qquad (3)$$

$$V_l = \frac{1}{2}y_i y_j k_{ij} = \frac{1}{2}a_i a_j\bar{k}_{ij}\cos^2(\omega t) \quad i, j = 1,2 \qquad (4)$$

$$V_{nl} = \frac{1}{2}y_i y_j y_k y_l b_{ijkl} = \frac{1}{2}a_i a_j a_k a_l\bar{b}_{ijkl}\cos^4(\omega t) \quad i, j, k, l = 1,2 \qquad (5)$$

where m_{ij}, k_{ij} and b_{ijkl} are respectively the general terms of the mass, the linear rigidity and the non-linear rigidity tensors in DB. $\bar{m}_{ij}$, $\bar{k}_{ij}$ and $\bar{b}_{ijkl}$ are respectively the general terms of the mass, the linear rigidity and the non-linear rigidity tensors in MB, whose expressions in terms of the mass and spring linear stiffenesses are given below.

The relationships between the expressions for the mass, the linear rigidity and the non-linear rigidity tensors in DB and MB can be obtained using the transition matrix [ϕ_{ij}] as (Z. Beidouri, M. EL Kadiri, and R. Benamar 2006).:

$$\begin{aligned} \bar{m}_{ij} &= \phi_{si}\phi_{tj}m_{st} \\ \bar{k}_{ij} &= \phi_{si}\phi_{tj}k_{st} \\ \bar{b}_{ijkl} &= \phi_{si}\phi_{tj}\phi_{pk}\phi_{ql}b_{stpq} \end{aligned} \qquad (6)$$

2.2.1 Expressions for the mass tensor in DB

The kinetic energy of the 2-dof system, exhibiting a harmonic motion, is:

$$T = \frac{1}{2}\omega^2\left(m_1A_1^2 + m_2A_2^2\right)\sin^2(\omega t) \qquad (7)$$

Eqs. (3) and (7) lead to the following expressions for the mass tensor in DB:

$$m_{11} = m_1, \quad m_{12} = m_{21} = 0, \quad m_{22} = m_2 \qquad (8)$$

2.2.2 Expression for the linear rigidity tensor in DB

For relatively small displacements, compared to the length of the bars, of the discrete geometricly symmetric system shown in fig. 1 (l_1=l_2=l_3=l), we can write :

$$\sin\theta_i = \frac{y_i - y_{i-1}}{l} \approx \theta_i \ , \quad i = 1,3 \qquad (9)$$

So that the linear potential energy of the 2-dof system results from the linear counterpart of the stretching forces in the four spiral torsional springs, which gives:

$$\begin{aligned} V_l &= \frac{1}{2}C_1\theta_1^2 + \frac{1}{2}C_2(\theta_2-\theta_1)^2 + \frac{1}{2}C_3(\theta_3-\theta_2)^2 + \frac{1}{2}C_4\theta_3^2 \\ &= \frac{1}{2l^2}\left[C_1y_1^2 + C_2(y_2-2y_1)^2 + C_3(y_1-2y_2)^2 + C_4y_2^2\right] \end{aligned} \qquad (10)$$

Assuming a symmetric rigidity matrix:

$$k_{ij} = k_{ji} \qquad (11)$$

Eqs. (4) and (10) lead to the expressions for the linear rigidity tensor :

$$\begin{aligned} k_{11} &= \frac{1}{l^2}(C_1 + 4C_2 + C_3) \\ k_{22} &= \frac{1}{l^2}(C_2 + 4C_3 + C_4) \\ k_{12} &= k_{21} = \frac{-2}{l^2}(C_2 + C_3) \end{aligned} \qquad (12)$$

2.2.3 Expression for the nonlinear rigidity tensor in DB

This work is a study of nonlinear vibration. In the study referenced (A. Eddanguir, Z. Beidouri, and R. Benamar 2009 ;

Z. Beidouri, A. Eddanguir, and R. Benamar, 7-9 July 2008). The nonlinearity was expressed by nonlinear cubic stiffness at the spiral torsional springs $M = C(\Delta\theta) + C^{nl}(\Delta\theta)^3$ and the longitudinal strains in the bar were neglected in the model. In this work, the linear rigidity of the spiral springs is taken into account, in order to represent the effect of the bending. The longitudinal deformations due to tension in the bars are represented in the model by stretching of axial springs having stiffnesses $k_i = (E_iS_i)/l_i$ in wich E_i (N/m^2) is Young's modulus of materials. S_i (m^2) is the cross-section and l_i (m) is the length of the bar i.
The potential energy of a spring longitudinal tension or compression is usually expressed as a function of longitudinal displacement along its axis (see Figure 2) (Δl_1 following the vector $\vec{v_1}$ for the spring of stiffness $k_1=(E_1S_1)/l_1$, Δl_2 following the vector $\vec{v_2}$ for the spring of stiffness $k_2=(E_2S_2)/l_2$ and Δl_3 following the vector $\vec{v_3}$ for the spring stiffness $k_3=(E_3S_3)/l_3$).

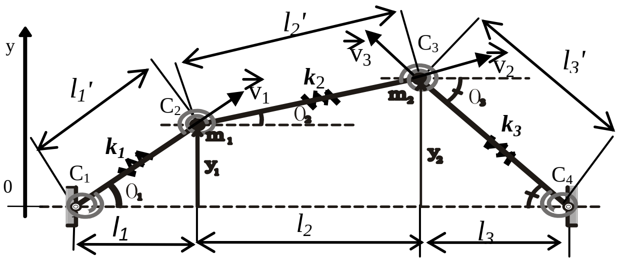

Figure 2: Illustration of the relationship between the transverse displacements y_1, y_2 and longitudinal deformations Δv_1, Δv_2, Δv_3 of the bars
Assume that the masses m_1 and m_2 are transversally displaced by y_1 and y_2, and let's calculate the resulting streching in the bars.
Applying the Pythagore's theorem to the triangle $l_1\, y_1\, l_1'$
And assuming a a relatively small displacements leads to :

$$l_1' = \sqrt{l_1^2 + y_1^2} = l_1(1+\frac{y_1^2}{l_1^2})^{1/2} = l_1(1+\frac{y_1^2}{2l_1^2}) \qquad (13)$$

The relationship existing between the longitudinal elongation Δl_1 folowing vector $\vec{v_1}$ of the bar number 1 and the transverse displacement y_1 of mass m_1 is

$$\Delta l_1 = l_1' - l_1 = l_1(\frac{y_1^2}{2l_1^2}) = \frac{y_1^2}{2l_1} \qquad (14)$$

The potential energy of the bar number one modeled by the axial spring of stiffness k_1 is given directly in terms of transverse displacement y_1 as follows:

$$U_1 = \frac{1}{2}k_1(\Delta l_1)^2 = \frac{1}{2}k_1\frac{y_1^4}{4l_1^2} = \frac{k_1}{8}\frac{y_1^4}{l_1^2} \qquad (15)$$

Applying similary the Pythagore's theorem to the triangle l_2 $(y_2\text{-}y_1)$ l_2' leads to:

$$l_2' = \sqrt{l_2^2 + (y_2-y_1)^2} = l_2(1+\frac{(y_2-y_1)^2}{l_2^2})^{1/2} = l_2(1+\frac{(y_2-y_1)^2}{2l_2^2}) \qquad (16)$$

Wich gives

$$\Delta l_2 = l_2' - l_2 = l_2(\frac{(y_2-y_1)^2}{2l_2^2}) = \frac{(y_2-y_1)^2}{2l_2} \qquad (17)$$

and

$$U_2 = \frac{1}{2}k_2(\Delta l_2)^2 = \frac{k_2}{2}\frac{(y_2-y_1)^4}{4l_2^2} = \frac{k_2}{8}\frac{(y_2-y_1)^4}{8l_2^2} \qquad (18)$$

For bar 3, one obtains similarly:

$$\Delta l_3 = l_3' - l_3 = l_3(\frac{y_2^2}{2l_3^2}) = \frac{y_2^2}{2l_3} \qquad (19)$$

$$U_3 = \frac{1}{2}k_3(\Delta l_3)^2 = \frac{1}{2}k_3\frac{y_2^4}{4l_3^2} = \frac{k_3}{8}\frac{y_2^4}{l_3^2} \qquad (20)$$

The non-linear potential energy of the 2-dof-system, due to the nonlinearity geometric non-lineaity wich appears when we express the potential energy of the three longitudinal springs due to transverse displacements. Assuming that ($l_1= l_2= l_3= l$) is given by:

$$V_{nl} = U_1 + U_2 + U_3 = \frac{1}{8l^2}(k_1+k_2)y_1^4 - \frac{1}{8l^2}(k_2+k_3)y_2^4 - \frac{k_2}{2l^2}y_1^3y_2 + \frac{3k_2}{4l^2}y_1^2y_2^2 - \frac{k_2}{2l^2}y_1y_2^3 \qquad (21)$$

Thus, expressing the potential energy of the longitudinal springs due to high transverse displacements shows to light a nonlinear form of potential energy as the latter is expressed in terms of $y_1^4, y_2^4, y_1^3y_2, y_1^2y_2^2$ *and* $y_1y_2^3$. This is a nonlinearity that comes from the geometry of the system. The purpose of the following section is to use expression (21) in order to determine the nonlinear rigidity tensor b_{ijkl}, usually encontred in similar studies. To do so the nonlinear potential energy is assumed to be written as: $V_{nl} = \frac{1}{2}y_iy_jy_ky_lb_{ijkl}$

The symmetry relationships usually encountered in the previous cases examined by the present method (M. EL Kadiri, R. Benamar and R. G. White, 2002a,b) are adopted here as follows:

$$b_{ijkl} = b_{ijlk},\quad b_{ijkl} = b_{klij},\quad b_{ijkl} = b_{ikjl}\quad i,j,k,l = 1,2 \qquad (22)$$

The tensor expression of the potential energy of the system can be written as

$$V_{nl} = b_{1111}y_1^4 + b_{2222}y_2^4 + 4b_{1112}y_1^3y_2 + 6b_{1122}y_1^2y_2^2 + 4b_{2221}y_1y_2^3 \qquad (23)$$

The identification of Eqs. (23) and (21) leads to the expressions for the non-linear rigidity tensor are in DB as:

$$b_{1111} = \frac{1}{8l^2}(k_1 + k_2); \quad b_{2222} = \frac{1}{8l^2}(k_2 + k_3)$$

$$b_{1122} = b_{2211} = b_{1212} = b_{2121} = b_{1221} = b_{2112} = \frac{k_2}{8l^2} \qquad (25)$$

$$b_{1112} = b_{1121} = b_{1211} = b_{2111} = -\frac{k_2}{8l^2}$$

$$b_{2221} = b_{2212} = b_{2122} = b_{1222} = -\frac{k_2}{8l^2}$$

The problem of nonlinear vibration of the system is fully described by the matrix m_{ij}, a linear rigidity matrix C_{ij} and nonlinear stiffness tensor b_{ijkl}.

2.3 *Model based on spectral analysis and Hamilton's principle and explicit procedure for solution*

2.3.1 *Model based on Hamilton's principle and spectral analysis*

In the present section, the non-linear amplitude equation is first established using Hamilton's principle and spectral analysis. Then, the procedure of solution, called the explicit prcedure, is presented for the non-linear vibration problem.

Applying Hamilton's principle to the vibration problem gives:

$$\delta \int_0^{2\pi/\omega} (V - T)dt = 0 \qquad (25)$$

Replacing T and V ($V=V_l+V_{nl}$) in this equation by their expressions given above, i.e. Eqs.(3) to (5), integrating the time functions, and calculating the derivatives with respect to the a_i's, leads to the following set of non-linear algebraic equations:

$$3a_i a_j a_k \bar{b}_{ijkr} + 2a_i \bar{k}_{ir} - 2a_i\omega^2 \bar{m}_{ir} = 0\,,\; i, j, k, r = 1,2 \qquad (26)$$

which can be written in matrix form as:

$$3\left[\bar{B}(a)\right]\{a\} + 2\left[\bar{K}\right]\{a\} - 2\omega^2\left[\bar{M}\right]\{a\} = \{0\} \qquad (27)$$

where ω^2 is the non-linear frequency parameter, the expression of which can be obtained by pre-multiplying Eq. (27) by $\{a\}^T$ from the left hand side, which leads to the following equation :

$$\omega^2 = \frac{a_i a_j \bar{k}_{ij} + \frac{3}{2} a_i a_j a_k a_l \bar{b}_{ijkl}}{a_i a_j \bar{m}_{ij}} \qquad (28)$$

The system (26) can then be written as:

$$3a_i a_j a_k \bar{b}_{ijkr} + 2a_i \bar{k}_{ir} - 2[(a_i a_j \bar{k}_{ij} + (3/2)a_i a_j a_k a_l \bar{b}_{ijkl}) / (a_i a_j \bar{m}_{ij})]\, a_i \bar{m}_{ir} = 0$$
$$r = 1,2 \qquad (29)$$

This is a non-linear algebraic system, identical formally to those derived previously in many case of continuous systems, which has to be solved numerically for determination of the non-linear mode shapes of the 2-dof system involving axial and transverse vibrations.

Three procedures have been succefly developed and used in previous works: Iterative procedure (A. Eddanguir, Z. Beidouri, and R. Benamar 2009; Z. Beidouri, A. Eddanguir, and R. Benamar 2008.), Explicit procedure (M. EL Kadiri, R. Benamar and R. G. White 2002a,b) and Linearised procedure (Z. Beidouri, M. EL Kadiri, and R. Benamar 2006). In this work, the second method is used.

2.3.2. *Explicit procedure*

This procedure of solution was initiated in Ref (M. EL Kadiri, R. Benamar and R. G. White, 2002a,b) for analysing the nonlinear vibrations of beams and fully clamped rectangular plates. Recently, the method has been slightly improved (Z. Beidouri, M. EL Kadiri, and R. Benamar. 2006). and applied to clamped simply supported clamped simply supported and clamped simply supported simply supported simply supported rectangular plates. A brief presentation of the method is made here.

This formulation is based on an approximation which consists on assuming, when dealing with first nonlinear mode that the contribution vector $\{a\}^T = \left[a_1\, a_2\right]$ can be written as $\{a\}^T = \left[a_1\, \varepsilon_2\right]$ stating that the second contribution ε_2 is very small versus a_1. This permits simplification of the nonlinear system by neglecting all of the nonlinear terms involving ε_2. Then, an approximated value of the frequency parameter can be obtained from the first approximate equation of system (26), i.e.:

$$3a_1^3\bar{b}_{1111} + 2a_1\bar{k}_{11} - 2a_1\omega^2\bar{m}_{11} = 0 \qquad (30)$$

which gives :

$$\omega^2 = \frac{\bar{k}_{11}}{\bar{m}_{11}} + \frac{3}{2}a_1^2\frac{\bar{b}_{1111}}{\bar{m}_{11}} \qquad (31)$$

Then, the second contribution ε_2 (a_2) is calculated explicitly from the second approximate equation of system (26):

$$\left(\bar{k}_{22} - \omega^2\bar{m}_{22}\right)\varepsilon_2 + \frac{3}{2}a_1^3\bar{b}_{1112} = 0 \qquad (32)$$

Substituting Eq. (31) into Eq. (32) leads to

$$\varepsilon_2 = -\frac{3a_1^3\bar{b}_{2111}}{2((\bar{k}_{11} + 3/2a_1^2\bar{b}_{1111})\bar{m}_{22}/\bar{m}_{11} - \bar{k}_{22})} \qquad (33)$$

Expression (33) is an explicit simple formula, allowing direct calculation of the second modal contribution to the first nonlinear mode shape, as function of the assigned first modal contribution a_1 and of the known parameters $\bar{m}_{rr}$, $\bar{k}_{rr}$ and $\bar{b}_{r111}$ (given below for treated systems). This defines the first nonlinear amplitude-dependent 2-dof system mode shape *y(a₁)* for a given assigned value of the first modal contribution like:

$$y(a_1) = a_1\vec{\phi}_1 + \frac{3a_1^3\bar{b}_{2111}}{((2\bar{k}_{11} + 3a_1^2\bar{b}_{1111}) - 2\bar{k}_{22})}\vec{\phi}_2 \qquad (34)$$

in which the predominant term, proportional to the first linear mode shapes $\vec{\phi}_1$, is $a_1\vec{\phi}_1$ and the other term, proportional to the other mode shape, i.e. $\varepsilon_2\vec{\phi}_2$, is the correction due to the nonlinearity for the first nonlinear mode shape.

To obtain the value of the nonlinear frequency, the multi-mode formula, written in MB has then been used (M. EL Kadiri, R. Benamar and R. G. White, 2002a,b):

$$\omega^2 = \frac{a_i a_j \bar{k}_{ij} + \frac{3}{2} a_i a_j a_k a_l \bar{b}_{ijkl}}{a_i a_j \bar{m}_{ij}} \qquad (35)$$

with $a_2 = \varepsilon_2$ for the first mode shape.

Similar steps are used for determination of the second nonlinear mode shape like a function of the contribution a_2. Using the approximation given below for the frequency parameter:

$$\omega^2 = \frac{\bar{k}_{22}}{\bar{m}_{22}} + \frac{3}{2} a_2^2 \frac{\bar{b}_{2222}}{\bar{m}_{22}} \qquad (36)$$

leads respectively to calculation of the first contribution and second nonlinear mode shape as:

$$\varepsilon_1 = -\frac{3a_2^3\bar{b}_{1222}}{2((\bar{k}_{22} + 3/2a_2^2\bar{b}_{2222})\bar{m}_{11}/\bar{m}_{22} - \bar{k}_{11})} \qquad (37)$$

$$y(a_2) = \frac{3a_2^3\bar{b}_{1222}}{((2\bar{k}_{22} + 3a_2^2\bar{b}_{2222}) - 2\bar{k}_{11})}\vec{\phi}_1 + a_2\vec{\phi}_2 \qquad (38)$$

The value of the nonlinear frequency is recalculated using the multi-mode formula (35), with $a_1 = \varepsilon_1$.

3. APPLICATIONS TO THE SYSTEM DESCRIBED IN SECTION (1)

For this system, the values of the masses, the length of the bars, the linear rigiditie of spiral and longitudinal springs are: $m_1=m_2=10$kg ; $l_1=l_2=l_3=1$m ; $C_1=C_2=C_3=C_4=10000$N/rad. E=210000MPa, S=7143mm^2; $k_1=k_2=k_3=150000$ N/m^2.
Since the masses are identical, the four torsional springs and the three longitudianl springs are identical, the present situation corresponds to a symmetric system.
In figure 3, the nonlinear frequency parameter is plotted versus the displacement amplitude of the first mass presenting the nonlinear mode dependences of the first and second mode shapes of the system examined. The nonlenear frequency parameters corresponding to the first and seconde nonlinear modes increases respectively 25 % and 45 % when the amplitude of vibration of mass 1 increases up 0.6. The nonlinearity effect is also largely observed.

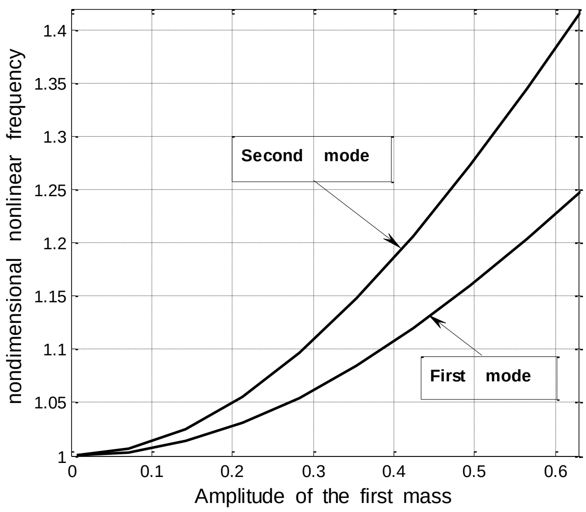

Figure 3**:** Non dimensional frequency parameter of system corresponding to the first and second nonlinear mode shapes.

In figure 4, the amplitude of the displacement of the second mass is plotted versus the displacement of the first mass presenting the first nonlinear mode of stadied system.

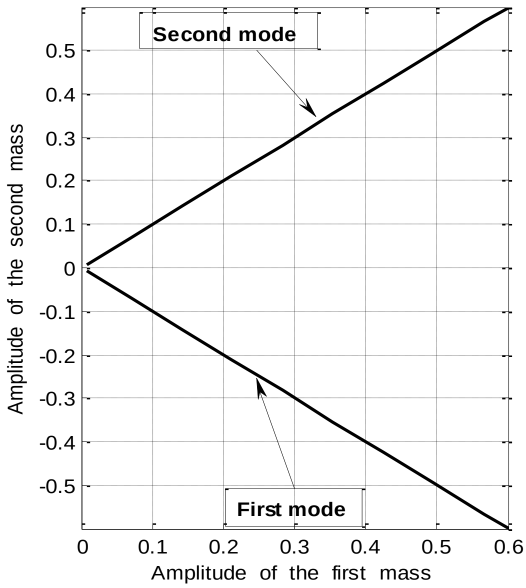

Figure 4**:** The second mass amplitude versus the first mass amplitude for system corresponding to the first nonlinear mode shape and the second nonlinear mode shape.

4. CONCLUSIONS

In this paper, a theory has been derived for the modelling of the geometrically nonlinear vibration of beams, based on a physical discretisation using both torsional spiral springs to represent the bending, and longitudinal springs to represent the axial vibrations induced by large ransverse displacement amplitudes. The corresponding mass, rigidity and nonlinear rigidity tensors have been expressed, leading to a nonlinear amplitude equation, similar to that encountred previously for structures of many types and geometry. The first results obtained are satisfactory and this model is suitable, with a higher degrees of freedom, to represent geometrically nonlinear vibratuins of beams with non uniform distributions of mass and stiffness and various boundary conditions

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