Saffman-Taylor Instability in Laponite and Mud

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Abstract We have studied the Saffman-Taylor instability for Laponite and mud. Our experiments showed several kinds of finger patterns which don't obey the classical theory of Saffman-Taylor instability. Our observations, on both finger patterns and rheology analyses results showed shear thinning behavior in high shear rates and shear thickening behavior in low shear rates for Laponite. This showed that the shear viscosity diverges at a point in which the shear viscosity has a bifurcation, this point has been identified. This result is in good agreement with the MRI rheology analysis results by Coussot et.al [2]. This is one of the important reasons that make Laponite such a complex fluid. We tested Darcy's Law for Laponite, we verified that the generalized Darcy's Law, suggested by Bonn et.al [5,11] is in good agreement with rheology analyses results in the shear thinning region of Laponite. Calculating the shear viscosity as a function of shear rate using generalized Darcy's Law on the fingers movements showed good agreement with rheology analyses results.
We also tested Mud; the finger patterns and Darcy's Law showed Newtonian behavior for this suspension. We also tested Darcy's Law for three Newtonian Fluids, detergent, oil and clothes conditioner, All three fluids resulted in a linear relationship between the fluids velocity and the pressure gradient in different rates of shear rates and pressure gradient. This finding supports the applicability of Darcy's Law in distinguishing Newtonian and non-Newtonian behavior in Fluids.

1. INTRODUCTION

The Saffman–Taylor instability has received much attention as an archetype of pattern forming systems, both theoretically and experimentally [1–3]. Most natural and industrial materials are non-Newtonian fluids. It is thus also important from a practical point of view to understand the instability in such 'complex fluids'. Lately, this instability has also been studied for non-Newtonian fluids, for which strikingly different fingering patterns are found. The physical origin of the very different structures is so far ill understood, mainly because most of these fluids exhibit multiple visco-elastic characteristics, which were not determined simultaneously. Very recently, fingering in yield stress fluids was studied both theoretically [4] and experimentally [5]. The theory on yield stress fluids has revealed that the Saffman–Taylor instability is modified drastically. Furthermore, we measure the applied pressure gradient during the viscous fingering experiment. For Newtonian fluids, Darcy's law gives the proportionality between the applied pressure gradient and the finger velocity. Darcy's law is the starting point of all theoretical treatments of the Saffman–Taylor instability. It is thus also important to study its applicability to non-Newtonian fluids, which we do in the present experiment.
We studied Saffman–Taylor instability for four Newtonian fluids, detergent, clothes conditioner, oil, and mud and also for two kind of non-Newtonian fluid, hair gel and Laponite. We have also examined the Darcy's law for these fluids. In this way, we have also studied the Rheological Properties of fluids.

2. EXPERIMENTS

We choose air as the fluid with lower viscosity and we do the experiments for detergent, clothes conditioner, oil, hair gel, Laponite and clay colloid suspended in detergent that briefly call it mud, as a more viscose fluid.
By dissolving Laponite powder in distilled water, Laponite suspension is achieved. Laponite powder includes particles of disk shape with 25 nm diameter and 1 nm thickness. Circular pages are negatively charged and the sides are positively charged. Mud is composed of clay particles with maximum diameter of 200 micrometers suspended in detergent, proximity of density of clay particles and detergent, prevents the deposition. We used mud with different concentrations; we pushed these fluids by compressed air with different pressure in Hele-Shaw cell, fig (1) shows the setup.

Fig1: Hele-Shaw cell

We filled the Hele-Shaw cell with the fluid and pushed it by compressed air with measured pressure flowed inside the cell. We used 25 fps CCD camera to process growing the fingers.

3. DAECY'S LAW

According to Darcy's Law we have the following relation between velocity, υ, and the pressure gradient, ∇p^{*},

$$v = \frac{-b^2}{12\eta}\nabla p \qquad (1)$$

b is the thickness of the cell. With certain pressure and velocity gradient we can derive fluid viscosity η. We studied stable fingers (viscous regime) in the rectangular cell. We can simply define ∇p^* as $p^*/\Delta x$, x is the distance between fingertips and the end of fluid, P* is also effective applied pressure. Finger velocity is calculated by the average velocity between two successive frames. By using continuity law and λ = d/w, in which d is the finger width and w is the cell width, we obtained fluid velocity as V = λv. By plotting the velocity vs. pressure gradient we can conclude η, constant η results in a Newtonian fluid, otherwise we have a non-Newtonian fluid.

4. RESULTS

4.1 Applicability of Darcy's Law:

We find out velocity vs. pressure gradient curve in the Hele - Shaw cell for each one of the mentioned fluids and Figs.(2&3) Show the results.
As we expected the curve of velocity vs. pressure gradient for detergent, clothes conditioner, and oil is linear and it means that they are Newtonian fluids, the mud's curve is also linear so it is a Newtonian fluid too, the type of its finger pattern also suggest mud as a Newtonian fluid(fig.5).
For hair gel and Laponite the relationship between velocity and pressure gradient is more rapid than linear, it means that we have shear thinning fluids (fig4).

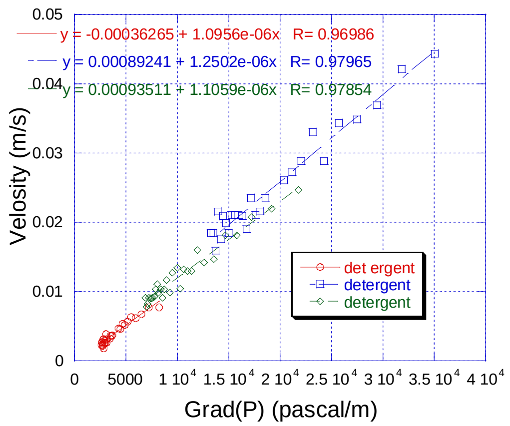


Figure 2-a Detergent

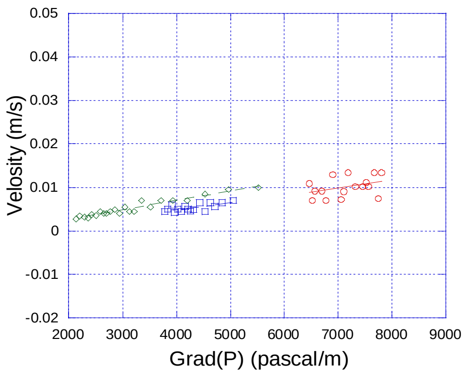


Figure2-b Oil

Fig.2: velocity vs. pressure gradient for two fluids, a) detergent, and b) oil. As can be seen they are all linear and so they are all Newtonian.

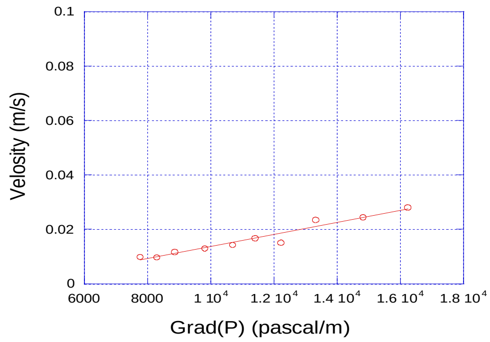


Figure3-a Cloth conditioner

Figure3-b Mud

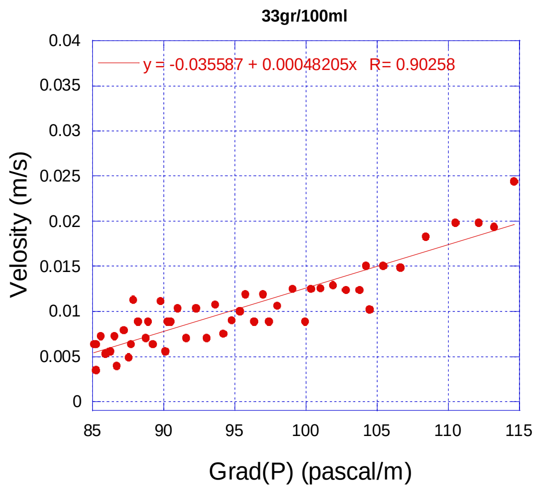
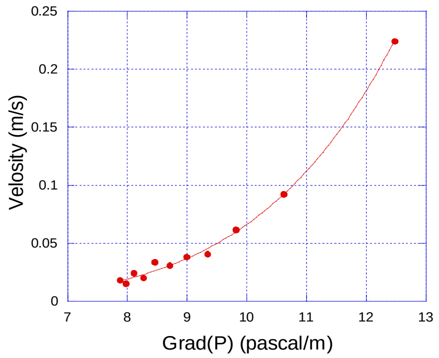
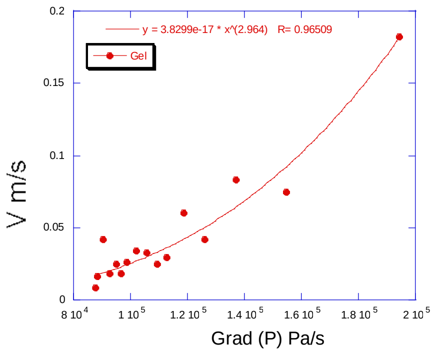


Fig.3: velocity vs. pressure gradient for two fluids, a), b) cloth conditioner, and b) mud. As can be seen they are all linear and so they are all Newtonian.



a) Laponite



b) Hair gel

Fig 4: velocity vs. pressure gradient for two fluids, a) Laponite, b) hair gel. They are all non-linear and so they are non- Newtonian.

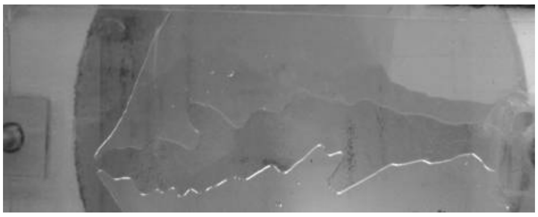
Bonn *et all*[5,11] suggested a generalized Darcy's Law for non-Newtonian fluid in which the viscosity supposed to be a function of shear rate:

$$v = \frac{-b^2}{12\eta(\dot{\gamma})}\nabla p \qquad (2)$$

In this fashion we can find the viscosity of a non-Newtonian fluid in any shear rate point from the $v-\nabla p$ curve using the generalized Darcy's Law. We have done this and compared the results with the rheology results; as we can see from the fig.7 the results are in good agreement with each other. So the $v-\nabla p$ curve is applicable for testing the kind of the fluids and the Darcy's Law (in addition to the generalized Darcy's Law) is applicable for determining their (variable) viscosity.

4.2 Viscose Fingering:

The type of viscose fingering was basically tip splitting for mud, which is similar to Newtonian fluids, and for Laponite was basically side branching, as it is the case for more non-Newtonian fluids. These results are in agreement with the Darcy's Law test of them. Fig (5) shows these results.


a) Laponite

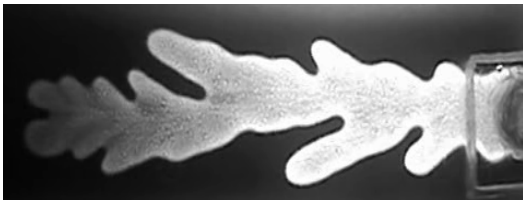

b) Mud

Fig 5: viscose fingers, a: Laponite, as can be seen it is basically side branched, and b: mud, tip splitting is clearly the case.

4.3 Rheology Measurements:

Laponite is a visco-elastic fluid and by using an mcR rheometer we measured its shear stress and shear viscosity in a wide range of shear rate (0.0001-1000 sec^{-1}), we have also measured its G', G" and complex viscosity, η^*, as we know η^* is related to G' and G" by the fallowing relation:

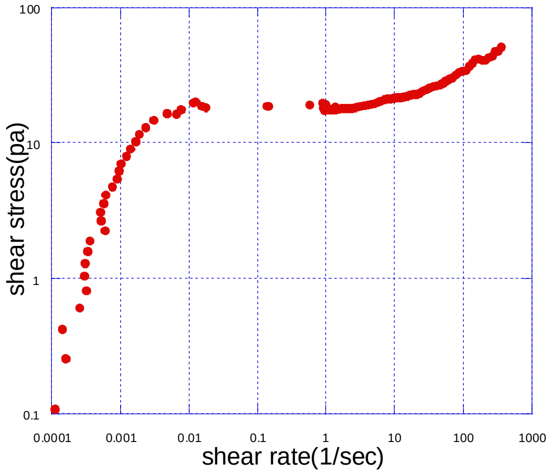
$$\eta^* = \frac{\left(G'^2 + G''^2\right)^{1/2}}{\omega} \qquad (3)$$

Fig.6 show shear stress vs. shear rate for Laponite, from fig.6-b it is clear that, in the shear rates higher than $0.1sec^{-1}$, it behaves like a shear thinning fluid. According to the Herchel-Balkly formula:

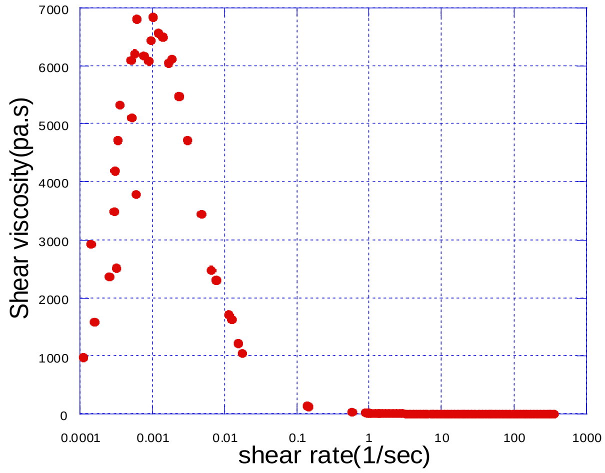
$$\sigma = \sigma_y + K\dot{\gamma}^{\alpha} \qquad (4)$$

In which σ is the shear stress, σ_y is the yield stress and α is the index which is less than unity for shear thinning fluids. As can be seen from the fit of the curve, the index is 0.56.

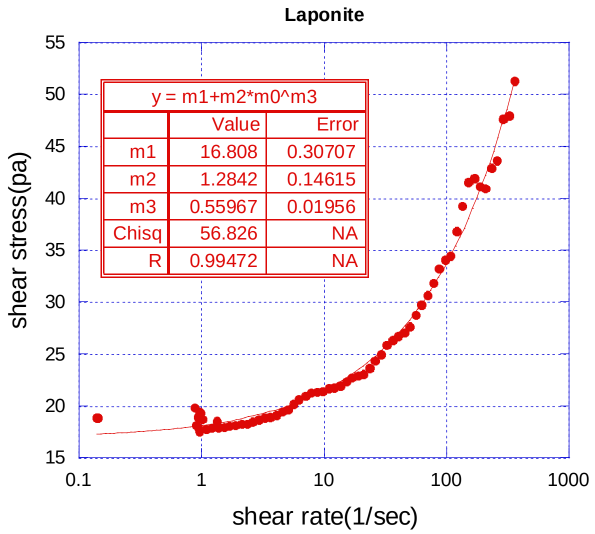
Fig.7 shows Shear viscosity vs. shear rate, as can be seen from fig.7-a, there is a divergence in the shear viscosity around $0.001sec^{-1}$ of shear rate , This result is in good agreement with the MRI rheology analysis results by Coussot *et.al* [2].



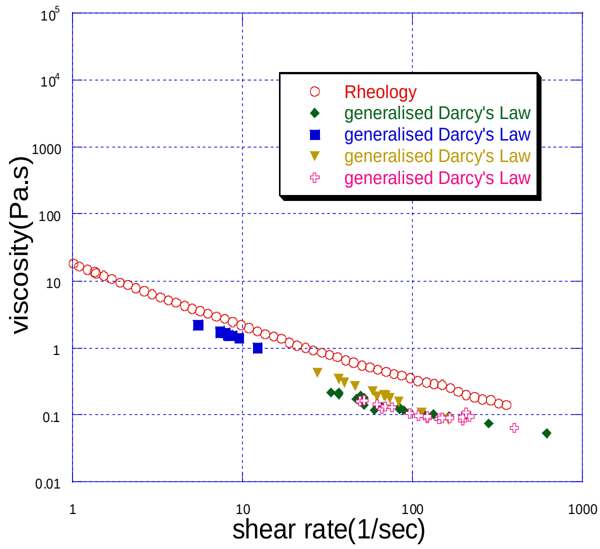
a) Whole Data



a)shear viscosity(rheology)



b) For high shear rates



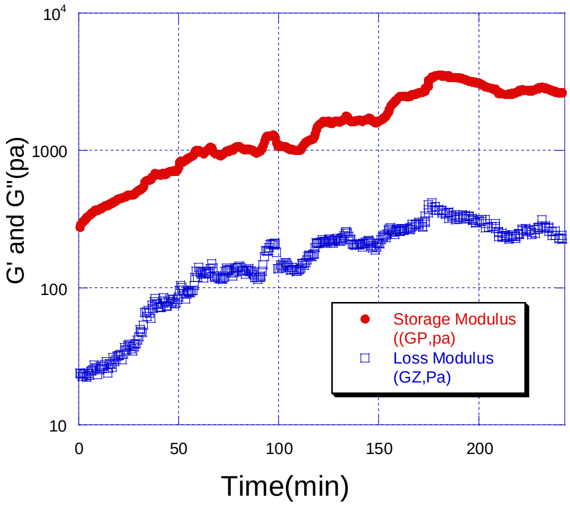
b) Coparision of rheology and the suggestion of the generalized Darcy's Law

Fig 6: shear stress vs. shear rate for Laponite. a) Whole results, and b) shear rates higher than 0.1sec^{-1}. The shear thinning behavior is obvious.

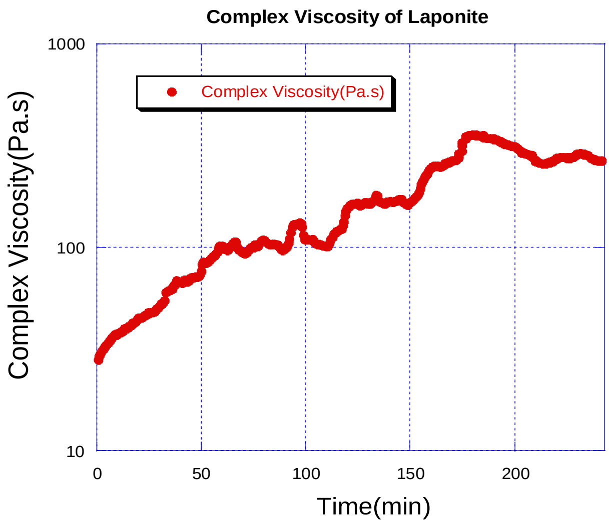
Fig 7: shear viscosity vs. shear rate for Laponite.; a)there is a convergent point in low shear rate in which viscosity has a bifurcation, b)the rheology results are in good agreements with the suggestion of generalized Darcy's Law.

Fig7-b compares the results of the rheology and the generalized Darcy's Law for shear viscosity, good agreement in the shear thinning region is apparent.

Fig.8 shows the G', G" and complex viscosity, η^*, as can be seen from the fig.8-b , Laponite has a considerable aging process.



a) G'and G"



b) Complex viscosity, η^*

Fig 8: a) G'and G", and b) complex viscosity η^*

5. CONCLUSION

We studied Saffman–Taylor instability for mud and Laponite. For Mud the finger patterns and Darcy's Law showed Newtonian behavior, but for Laponite both of them showed non-Newtonian behavior, shear thinning behavior in high shear rates and shear thickening behavior in low shear rates. These results showed that the shear viscosity diverges at a point in which the shear viscosity has a bifurcation, this point has been identified. This result is in good agreement with the MRI rheology analysis results by Coussot et.al [2]. We verified that the results of the generalized Darcy's Law, suggested by Bonn et.al [5,11] is in good agreement with rheology analyses results in the shear thinning region of Laponite.

Our finding supports the applicability of Darcy's Law in distinguishing Newtonian and non-Newtonian behavior in Fluids, all three Newtonian Fluids, detergent, oil and clothes conditioner, and also the mud, resulted in a linear relationship between the fluids velocity and the pressure gradient in different rates of shear rates and pressure gradient, while the two non-Newtonian fluids, hair gel and Laponite, resulted in a non-linear relationship.

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