Analysis of Four Element Chua Circuit Containing New Passive Component "Memristor"

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Abstract: After being realized practically in 2008, the scientists have started investigating the applications of the memristor element in different engineering fields. In this paper, hoping to be helpful in the modelling of real biomedical systems and the design of secure communication systems, the four-element Chua circuit containing the memristor is examined and analyzed. When the parameters of the circuit are changed, various dynamic behaviour sequences such as limit cycle oscillations and chaotic attractors are obtained.

Keywords: Memristor; Menductance; Memristance; Chaotic behaviours; Chua circuit.

1. INTRODUCTION

The recent engineering studies on artificial intelligence and neuroscience have shown that the behaviours of synapses, the connections between neurons, exhibit similar characteristics with those of the memristor. These studies prove that the use of the memristor for modelling a synapse is the best known method at the moment. (Adee, 2008). The application of the memristor as a synapse is shown in Fig.1. On the other hand, it is known that the neural systems and the similar natural systems exhibit chaotic behaviours. Furthermore, the high frequency chaotic oscillations have immense potential for applications in secure communication (Roy *et al.*, 2003). One possibility for obtaining high frequency chaotic circuits is to use nanoscale devices, like the memristor (Sudheer *et al.*, 2009; Muthuswamy, 2009a). Therefore, the memristor as the fourth passive component and its applications have gained an increasing attention on the areas of computer, biomedical and communication (Jo *et al.*, 2010; Kim *et al.*, 2010; Laiho *et al.*, 2010; Riaza, 2010).

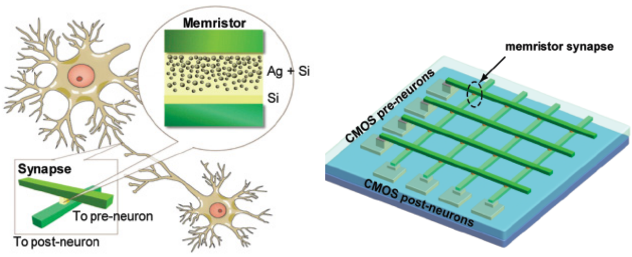


Fig.1. Nanoscale application of the memristor as a synapse. (a) Schematic illustration of the concept of using memristors as synapses between neurons. (b) Schematic of a neuromorphic with CMOS neurons and memristor synapses in a crossbar configuration (Jo *et al.*, 2010).

The definition of the passive circuit components used in electrical circuit theory are based on the relations among the four circuit variables named current (i), voltage (v), charge (q) and flux (φ). The relation between voltage and current ($dv=Rdi$) defines the *resistor*, the relation between charge and voltage ($dq=Cdv$) defines the *capacitor* and the relation between flux and current ($d\varphi=Ldi$) defines the *inductor* (Chua, 1969; Ho *et al.*, 2009). The basic electrical properties of these components are represented by *resistance* (R), *capacitance* (C) , and *inductance* (L), respectively. They have been called as "three basic passive circuit elements" for a hundred years.

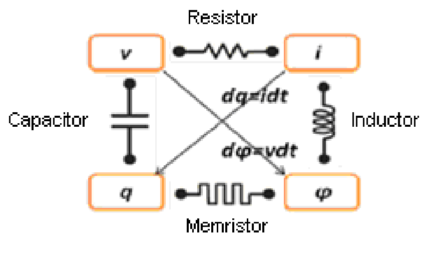
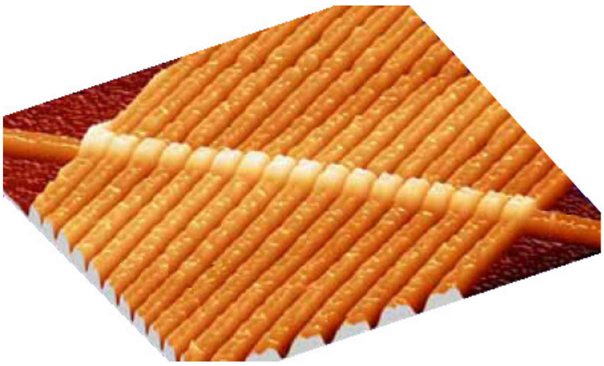


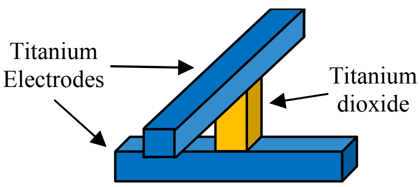
Fig. 2. Four basic passive circuit components (Ho *et al.*, 2009).

Considering the relation between the flux and charge ($d\varphi=Mdq$), Chua pointed out the existence of the fourth passive circuit component in 1971 (Chua, 1971). By definition of this component, known as memory resistor (memristor), all the relations among the four basic circuit variables have been completed fully as shown in Fig. 2. It can be observed from this figure that the relations between the basic variables are bilateral. If the flux-charge relation is considered for the memristor, the electrical property of the element is defined as *memristance* (M) which has the same unit with the resistance. On the contrary, if the charge-flux relation is considered, the electrical property of the memristor is described as *menductance* (W). Menductance has similar properties with conductance which is the inverse of the resistance (Ho *et al*., 2009).

After Chua's work in 1971, it had been appeared only a few works in the literature for a long time since it was thought that the memristor was only a theoretical element and it could not be realized practically. This continued until 2008 when a group of scientists from HP Labs reported that the memristor component had been fabricated using nanotechnology. (Strukov et al., 2008; Yang et al., 2008). This component was produced by applying titanium oxide (TiO_2) between titanium contacts as shown in Fig. 3. There has been appeared a great concern about the fourth passive component since its production in the laboratory conditions. The most important reason of this concern is the realizations of some applications (Johnson, 2008; Tour *et al*., 2008; Witrisal, 2009; Riaza *et al*., 2010; Robinet *et al*., 2010) which are potential for the developments of the more powerful computers operating like human brain. They offer an enabling low cost technology for *non-volatile* memories where future computers would turn on instantly without the usual "booting time", currently required in all personal computers (Itoh *et al*., 2008). All of these applications reveal that the research on the memristor is a subject that will be insisted on it in the near future.



(a)



(b)

Fig. 3. a) Atomic force microscope image of the 1×17 array of 50 × 50 nm^2 memristor. b) Schematic illustration of the connections of titanium electrodes with titanium oxide (Williams, 2009).

New research results have been appeared on the generation and analysis of chaotic signals using memristor element in the last few years. They are based on the replacement of the nonlinear resistance in the Chua circuit, which is the most basic chaos generating circuit (Kennedy, 1993) containing two capacitors, one inductor, one linear and one nonlinear resistor, by the memristor. There are many other variations of the Chua's circuit appearing in the literature used to obtain different forms of the chaotic behaviour. One of them is the *four element Chua* (4EC) *circuit* obtained by removing the linear resistor in the basic Chua circuit (Itoh *et al*., 2008; Muthuswamy, 2009a). The research studies on the 4EC circuit is limited on the area whether this circuit would generate chaotic behaviour due to less number of components used with respect to the conventional Chua circuit. In this paper, it is shown that the different forms of nonlinear behaviour sequences in the 4EC circuit can be obtained by changing the value of any element. When the value of the inductance L is increased or the value of the capacitance C_1 are decreased in the 4EC circuit, it can be observed the equilibrium point, period-doubling sequences, spiral Chua attractor and double scroll chaotic attractor behaviours, respectively. These behaviours can be used for the purpose of modelling some dynamics in the nature.

The remainder of the paper is organised as follows. In Section 2, we briefly recall the basic properties of the memristor element defined with respect to the menductance characteristics. Section 3 and 4 outline the four element Chua circuit with Chua diode proposed by Barboza and Chua (2008) and with memristor introduced by Muthuswamy (2009a), respectively. Also, various nonlinear behavioural sequences in the 4EC circuit with a flux-controlled memristor by changing the element values in the circuit are obtained in Section 4. Finally, some conclusions are given in Section 5.

2. MEMORY RESISTOR (MEMRISTOR)

Definition 2.1. A *memristor* can be defined as a two-terminal nonlinear passive and resistive element whose value varies in dependence of its magnetic flux or electrical charge. The relations between the terminal voltage (v) and the terminal current (i) of a memristor are expressed by

$$i = W(\varphi)v \tag{1}$$

and

$$v = M(q)i \tag{2}$$

where the nonlinear functions $W(\varphi)$ and $M(q)$ are named as menductance and memristance, respectively (Itoh *et al*., 2008). They are defined by

$$W(\varphi) = dq(\varphi)/d\varphi\,, \tag{3}$$

$$M(q) = d\varphi(q)/dq\,. \tag{4}$$

These equations show that the menductance is a flux controlled and the memristance is a charge controlled electrical property. In this work, only the flux controlled memristor given by (3) is considered. The menductance of the memristor changes according to the flux generated on it. While the flux generated in one direction increases the menductance (that is, it increases the conductance and decreases the resistance), the flux in the opposite direction decreases the menductance (that is, it decreases the conductance and increases the resistance). If the circuit is disenergized, the memristor remembers its last menductance value and it remains at this value until the energy is resupplied. The purpose of the word "memory" in the name of the memory resistance is that the value of the menductance function at an instant t_0 depends on the time integral of the voltage across the component from t=-∞ to $t=t_0$, and hence the menductance value is related with the whole history of the component (Chua, 1971). This behaviour is called *memristive* behaviour.

Consider the nonlinear φ-q characteristic of a flux controlled memristor given as the following expression:

$$q(\varphi) = \alpha\varphi + \beta\varphi^3 . \qquad (5)$$

Instead of (5), its piecewise linear approximated form

$$q(\varphi)\big|_{pwl} = d\varphi + \frac{1}{2}(c-d)\big(|\,\varphi + x\,| - |\,\varphi - x\,|\big) \qquad (6)$$

will be used in the simulations for the sake of simplicity in the calculations and realizations by op-amp circuits. Based on (5) and (6), two different menductance definitions can be made (Itoh *et al.*, 2008):

1. Type-I menductance: All of the parameters (α, β; c, d, x) of (5) and (6) are chosen as real and positive.
2. Type-II menductance: The parameters of α and c are selected as real and negative, β, d and x are chosen as real and positive.

The characteristics for both types of menductances given by (5) and (6) are shown in Fig. 3. Also, the menductance function is obtained by the use of (3) as

$$W(\varphi) = \alpha + 3\beta\varphi^2 \qquad (7)$$

$$W(\varphi)\big|_{pwl} = \begin{cases} c, & |\varphi| < x \\ d, & |\varphi| > x \end{cases} \qquad (8)$$

The characteristic diagrams of (7) and (8) are shown in Fig. 4. Similar characteristic curves can be obtained for charge-controlled memristor. In this paper, we use piecewise linearly approximated menductance function given in (8).

The circuit diagrams necessary for the realization of the memristor by the use of op-amps are given in Muthuswamy (2009b) at length.

3. FOUR ELEMENT CHUA (4EC) CIRCUIT

Although many chaotic systems are invented after Lorenz's work opening new horizons in 1963 (Lorenz, 1963), it had been hardly possible to provide a rigorous proof of chaos exactly in these systems until Chua's circuit which has proved the existence of chaos rigorously for the first time (Matsumoto *et al.*, 1985; Chua, 1992). According to the Shilnikov and Poincare-Bendixson theorems (Khalil, 2001), an autonomous continuous-time dynamical system should be at least third dimensional, that is it should have at least three energy storage components, for being chaotic. Barboza and Chua in 2008 proposed a circuit simpler than the classical Chua's circuit, namely four element Chua (4EC) circuit (Barboza *et al.*, 2008). Since this circuit as shown in Fig. 5 contains the minimum requirements for chaos, it is named as the simplest autonomous chaotic circuit (Muthuswamy, 2009a).

The 4EC circuit is defined by the following three state equations:

$$\frac{dv_1}{dt} = \frac{1}{C_1}\big(i - f(v_1)\big), \qquad (9)$$

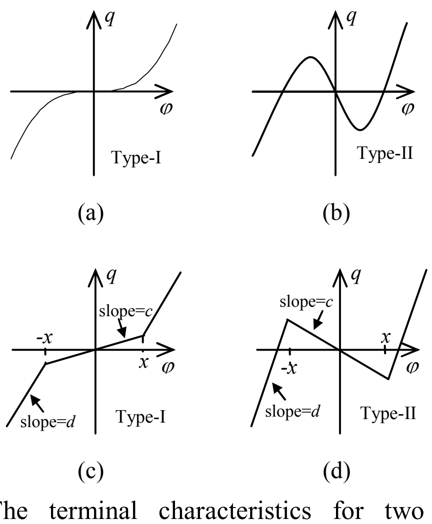


Fig. 3. The terminal characteristics for two types of menductance functions; a,b) the actual c,d) piecewise linearly approximated.

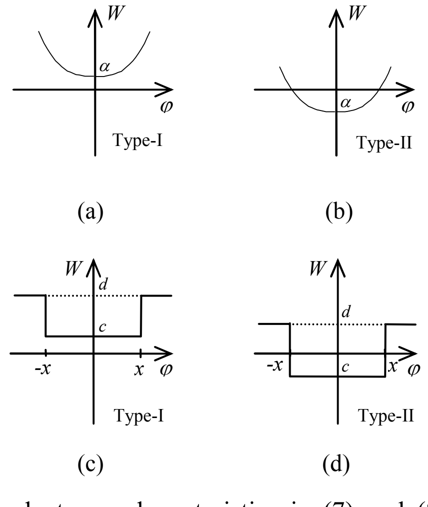


Fig. 4. Menductance characteristics in (7) and (8) for two types; a,b) the actual, c,d) piecewise linearly approximated.

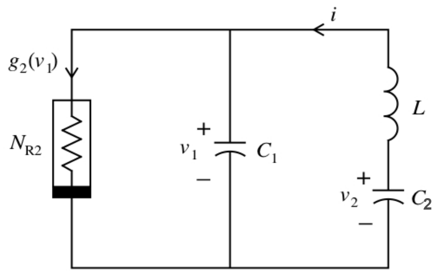


Fig. 5. The 4EC circuit.

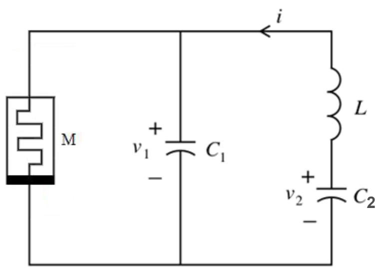


Fig. 6. The 4EC Circuit with a memristor.

$$\frac{di}{dt}=\frac{1}{L}\left(v_1-v_2\right), \tag{10}$$

$$\frac{dv_2}{dt}=\frac{1}{C_2}i\,. \tag{11}$$

Here, $f(.)$ represents the piecewise linear characteristics of the resistor N_R defined by

$$f(v_1)=G_b v_1+\frac{1}{2}(G_a-G_b)(|v_1+E|-|v_1-E|)\,, \tag{12}$$

where $E>0$, $G_a<0$, $G_b>0$. Muthuswamy (2009a) pointed out that this circuit generates the chaotic behaviour in the type of double scroll attractor for the parameter values C_1=33 nF, C_2=12 nF, L=83.3 mH, G_a=-500 µs, G_b=933.3 µs and E=1 V.

4. THE 4EC CIRCUIT WITH MEMRISTOR

Fig. 6 shows the 4EC circuit where the nonlinear resistance is replaced by a memristor. The state equations of this circuit are obtained as

$$\frac{dv_1}{dt}=\frac{1}{C_1}\left(i-W(\varphi)v_1\right), \tag{13}$$

$$\frac{di}{dt}=\frac{1}{L}\left(v_1-v_2\right), \tag{14}$$

$$\frac{dv_2}{dt}=\frac{1}{C_2}i \tag{15}$$

From (2) and (4), the state variable v_1 is defined by

$$v_1=M(q)i=\frac{d\varphi}{dq}\frac{dq}{dt}=\frac{d\varphi}{dt} \tag{16}$$

The φ-q characteristic of the memristor is chosen as in Fig. 3d where the values are x=1.5x10^{-4}, c=-0.5x10^{-3} and d=4.33x10^{-3}. Therefore, the menductance function $W(\varphi)$ which is obtained by (3) is shown in Fig. 7. This function is simpler than the function used in (Muthuswamy, 2009a). In spite of its simplicity, this menductance function still generates the double scroll attractor for the element values C_1=33 nF, C_2=12 nF and L=83.3 mH.

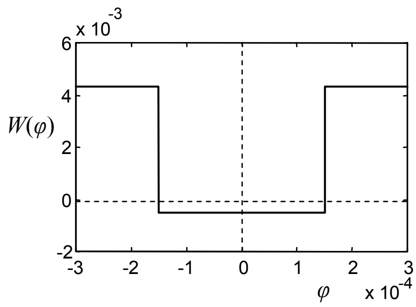
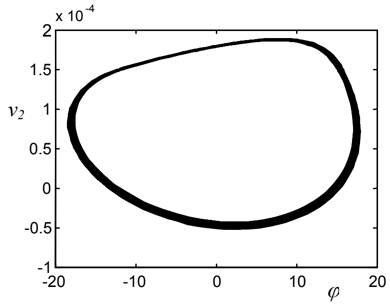


Fig. 7. Piecewise linear menductance characteristic for the 4EC circuit with the memristor.

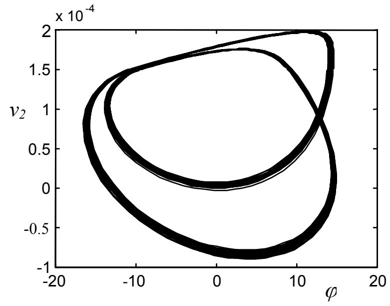
Figs. 8 and 9 show the simulation results for the 4EC chaotic circuit with memristor. It can be observed from Fig. 8 that the circuit exhibits different dynamical behaviours when the values of all components, except that of the capacitance C_1, in the circuit are kept constant. The initial values associated with the energy storage elements' states are chosen as $[v_1,\ i,\ v_2]|_{t=0s}$ = [0.1, 0, 0]. Since the resulting equilibrium points are stable for sufficiently large values of the capacitance C_1 (C_1>65nF), the system displays a stable equilibrium point behaviour depending on the initial conditions of the elements. While the trajectories approach to the equilibrium point slowly for the value of 65 nF, they approach to the equilibrium point more rapidly for larger values of C_1 (for example 600 nF). When the value of C_1 is reduced slightly lower than 65 nF, the stable equilibrium points change to unstable states and the system generates various behaviours called limit-2 cycle (48 nF), limit-4 cycle (31 nF), spiral chaotic attractor (44 nF) and double scroll chaotic attractor (31 nF). When the capacitance is reduced below 31 nF, the system behaviour becomes completely unstable.

In a manner similar to changing the value of C_1, when the values of all components, except that of the inductance L, in the circuit are kept constant, the circuit exhibits different behaviours for changing values of L. Fig. 9 indicates some of the selected behaviours showing the resulting chaotic sequences.

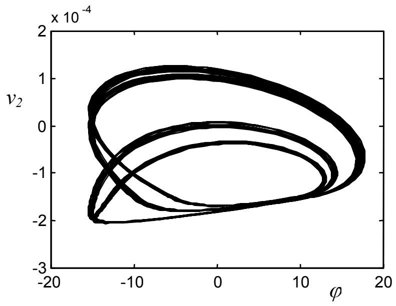
Since only the steady-state responses are considered in illustrating the chaotic behaviours, the transient responses are eliminated while the behavioural sequences in this



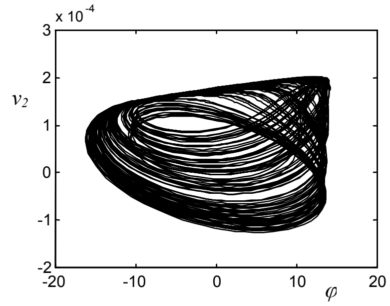
(a) Limit-1 cycle



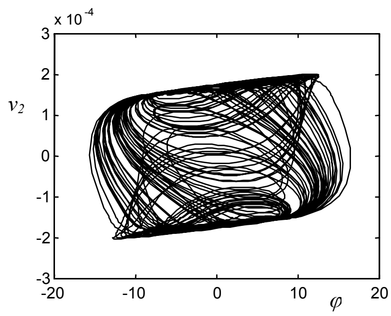
(b) Limit-2 cycle



(c) Limit-4 cycle

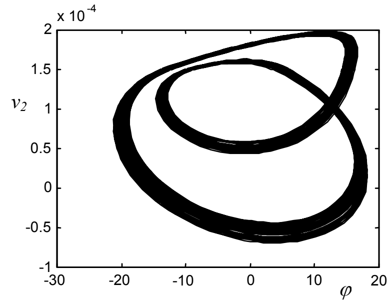


(d) Spiral chaotic attractor

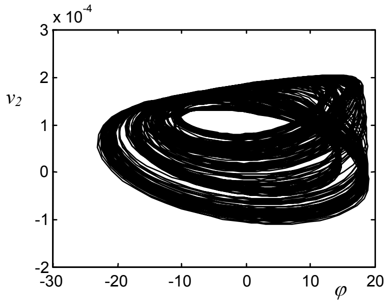


(e) Double scroll chaotic attractor

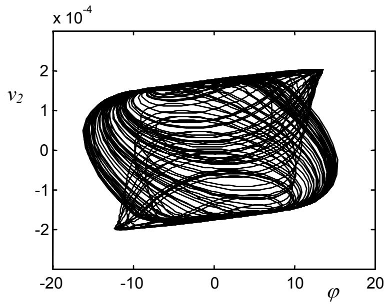
Fig. 8. The chaotic behaviour sequence of the 4EC circuit with memristor for a) C_1=59 nF, b) C_1=48 nF, c) C_1=46 nF, d) C_1=44 nF, e) C_1=31 nF.



(a) Limit-2 cycle



(b) Spiral chaotic attractor



(c) Double scroll chaotic attractor

Fig. 9. Some chaotic behaviours of 4EC circuit with memristor changing with varying inductance values; a) L= 15 mH, b) L= 17 mH, c) L= 75 mH.

section are obtained. The simulation results are given by using the differential equation solver *ode*23 of MATLAB. To achieve this, the simulation time and the tolerance values are chosen as 40 ms and $AbsTol = RelTol = 1\text{x}10^{-4}$, respectively.

5. CONCLUSIONS

In this paper, various nonlinear behavioural sequences in the 4EC circuit whose Chua's diode are replaced with a flux-controlled memristor are obtained by changing the element values in the circuit. It is clear that the findings and results obtained in this paper have a good potential in view of understanding, manipulating and generating systematic theories for the real chaotic systems such as encryption and secure communication, biomedical, laser and some other engineering systems. Clearly, the basic ideas and approach using the memristor element presented in this paper can be applied to the similar variants of the Chua circuit and the other chaotic systems for investigation.

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