Why You Should Consider Nature-Inspired Optimization Methods in Financial Mathematics

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Especially after the success of genetic algorithms (GAs), which is influenced by the natural selection phenomenon, the last two decades have witnessed an increasing emphasis in the computer science and engineering communities on the studies about nature-inspired computing. A considerable number of algorithms mimicking some phenomena in nature have yielded a wide spectrum of applications. Algorithms in this class (genetic algorithm, genetic programming, differential evolution, particle swarm optimization, ant colony optimization, etc.), which have been developed particularly for complicated multidimensional continuous and combinatorial optimization problems, together with a literature review of their applications in financial mathematics (particularly application to the portfolio optimization problem and its derivatives) constitute the main theme of this study.

Keywords**:** financial mathematics, nature-inspired methods, metaheuristics, optimization, portfolio optimization.

1. INTRODUCTION

Financial Mathematics is a flourishing area of modern science. The subject has developed rapidly into a substantial body of knowledge since the days of pioneering people of this discipline such as Black, Scholes and Merton. As of today, numerous applications of financial mathematics have become vital for the financial institutions, especially as regards to trading, asset management, and risk control of complicated financial positions.

The basic mathematics that underlies the subject is probability theory, with its strong connections to partial differential equations and numerical analysis. On the finance side, the main topics of importance are the pricing of derivatives, the evaluation of risk, and the management of portfolios. In fact, in today's world, many aspects of capital markets management are becoming more quantitatively and computationally sophisticated; however it is still a valid argument to say that everything began with derivatives.

As complicated as the problems in consideration get, or as complicated as the approaches/models for handling of these problems get, conventional tools or methodologies become insufficient. In this paper, we will try to focus on the portfolio optimization problem, which is one of the main topics in financial mathematics. We will try to identify why and when conventional methods become insufficient, and metaheuristics (especially nature-inspired optimization methods) might constitute a remedy. We will also mention the major studies in the literature on this topic.

The organization of the paper is as follows: After this introductory section, we will try to revisit the definition of the portfolio optimization problem with existing models in the literature. In Section 3, we will give brief descriptions of some popular nature-inspired optimization algorithms. Section 4 is nothing but a condensed literature review about the application of the nature-inspired methods for the solution of the portfolio optimization problem. In Section 5, we will try to give our concluding remarks.

2. PORTFOLIO OPTIMIZATION

Portfolio is nothing but the allocation of wealth (or resources in hand) among several assets. Portfolio optimization, which addresses the ideal assignment of resources to existing assets, has been one of the important research fields in modern risk management, or more generally financial management.

A fundamental answer to this problem was given by Markowitz (1952, 1959), who proposed the mean-variance model, which is now considered as the basis of modern portfolio theory. In Markowitz's approach, the problem was formulated as an optimization problem with two criteria:

- the profit (sometimes also referred to as reward or return) of a portfolio (measured by the mean) that should be maximized, and
- the risk of the portfolio (measured by the variance of return) that should be minimized.

In the presence of two criteria, there is not a single optimal solution to the problem (i.e. a single optimal portfolio), but a set of optimal portfolios. Certainly, there is a trade-off between risk and return.

Since the mean-variance theory of Markowitz, research has been performed about extending or modifying the basic model in three directions (Anagnostopoulos *et al.* 2010):

(i) the simplification of the type and amount of input data,

(ii) the introduction of alternative measures of risk, and

(iii) the incorporation of additional criteria and/or constraints.

In the following subsections, we will try to summarize the basic models extending that of Markowitz while trying to identify the differences.

2.1 Mean-Variance Model

Markowitz's mean-variance model, in which the variance or the standard deviation is considered as a measure of risk, has been regarded as a quadratic programming problem. In spite of its popularity during the past, the mean-variance model is based upon the assumptions that an investor is risk averse and that either:

(i) the distribution of the rate of return is multivariate normal, or

(ii) the utility of the investor is a quadratic function of the rate of return (Chang *et al.* 2009).

However, neither (i) nor (ii) holds in practice, unfortunately. It is now widely recognized that the real world portfolios do not follow a multivariate normal distribution. Many researchers suggested that one cannot blindly depend on mean-variance model. That is why various risk measures such as semi-variance model, mean absolute deviation model and variance with skewness model have been proposed.

2.2. Semi-variance model

With this model, eventually the variance component of the Markowitz's quadratic objective function can be replaced by other risk functions such as semi-variance. With an asymmetric return distribution, the mean-variance approach leads to an unsatisfactory prediction of portfolio behavior. Indeed, Markowitz himself suggested that a model based on semi-variance would be preferable.

2.3. Mean absolute deviation model

Konno and Yamazaki (1991) were the ones who first proposed a mean absolute deviation portfolio optimization model as an alternative to the Markowitz mean-variance portfolio selection model, with the advantage of the portfolio selection problem to be formulated and solved via linear programming. It has been shown that this model yields similar results to the mean-variance model. Moreover, due to its simplicity, computational it outperforms to the mean-variance model (Konno 2003, Konno and Koshizuka. 2005).

2.4. Variance with skewness

Samuelson (1958) was the one who first noticed the importance of the third order moment in portfolio optimization. A portfolio return may not be a symmetric distribution. The distribution of individual asset returns tends to exhibit a higher probability of extreme values than is consistent with normality.

In order to capture the characteristics of the return distribution and to provide further decision-making information to investors, this model includes skewness into the mean-variance model. Although the existence of skewness in portfolios has been demonstrated many times, only a few studies to date have proposed incorporating skewness into the portfolio optimization problem (Chang *et al.* 2009). Konno and Yamamoto (2005) showed that a mean-variance skewness portfolio optimization model can be solved exactly in a fast manner by using the integer programming approach.

3. NATURE-INSPIRED OPTIMIZATION METHODS

Nature-inspired optimization methods fall into the class of metaheuristics. These are nothing but some methods influenced by the existing behaviors/phenomena for the solution of an optimization-like problem in nature. A very simple source of inspiration is the behavior of a colony or a swarm while searching for food.

In computer science, the term metaheuristic is used to describe a computational method, which optimizes a problem by iteratively trying to improve a candidate solution with regard to a given measure of quality. In other words, such methods are nothing but systematical trial-and-error approaches. Metaheuristics (sometimes also referred to as derivative-free, direct search, black-box, or indeed just heuristic methods) make few or no assumptions about the problem (such as modality or dimension) being optimized and can search very large spaces of candidate solutions. Moreover, most of these algorithms by definition are easily adaptable to parallel computing, which makes them applicable in very large-scale problems.

However, it should be noted that metaheuristics do not guarantee an optimal solution is ever found. On the other hand, for each algorithm, numerous studies (most of which are empirical) have been carried out in order to understand how the algorithm parameters should be adjusted for increasing the success probability.

Originally, metaheuristics were proposed for combinatorial optimization in which the optimal solution is sought over a discrete search-space. An example is the traveling salesman problem, where the search-space of candidate solutions grows exponentially as the size of the problem increases which makes an exhaustive search for the optimal solution infeasible. Popular metaheuristics for combinatorial problems include simulated annealing (Kirkpatrick *et al.* 1983), tabu search (Glover 1989, 1990), genetic algorithms (Holland 1975), and ant colony optimization (Dorigo 1992, Dorigo *et al.* 1996, 1997, 1999).

Later, metaheuristics for problems over real-valued search-spaces were also proposed. In such problems, the conventional approach was to derive the gradient of the function to be optimized, and then to employ gradient descent or a quasi-Newton method. Metaheuristics do not use the gradient or Hessian matrix; hence their advantage is that the function to be optimized need not be continuous or differentiable; moreover, it can also have constraints. Popular metaheuristic optimizers for real-valued search-spaces include particle swarm optimization (Kennedy and Eberhart 1995), differential evolution (Storn and Price 1997) and evolution strategies (Rechenberg 1971, Schwefel 1974).

All algorithms of this sort were initially proposed for single-objective problems. However, throughout the years, multi-objective extensions of these algorithms have been proposed. One of the early attempts was the extension of simulated annealing to multi-objective problems by Czyzak and Jaszkiewicz (1998). Another one was that of Hansen (2000) for extension of tabu search. In a review article, Ehrgott and Gandibleux (2000) listed a bibliography of multi-objective optimization approaches for combinatorial problems (not only considering metaheuristics, but also the conventional approaches). In another review article, Coello (2006) identified the historical development of the research studies about extending these algorithms to multi-objective problems. This review was not limited to the combinatorial problems; on the other hand, only the studies regarding population-based metaheuristics were considered.

In this paper, we will focus on the nature-inspired metaheuristics and their applications to the portfolio optimization problem. In the upcoming subsections, we will briefly summarize the most popular and well-known ones.

3.1 Genetic Algorithm

Influenced from the "survival of the fittest" principle in the evolution theory, Holland (1975) proposed genetic algorithms for the solution of combinatorial optimization problems. The method simply relies on representation of the solution candidates by means of chromosomes, via which the relevant objective function is evaluated. The solution candidates constitute a population, which will be evolved throughout the generations (with the programmer's perspective, the generations correspond to the populations in succeeding iterations). Performing these evaluations and considering the fitness of each solution candidate (i.e. the value of the objective function corresponding to that candidate), it is decided which candidates deserve to survive and to be transferred to the next generation. Certainly, as in the evolution process, diversity is added by means of some operators such as crossover (yielding the hybridization of high-quality solution candidates) and mutation.

Even though genetic algorithm was originally proposed by Holland for single-objective combinatorial problems, later it has been extended to real-valued optimization problems; even to multi-objective optimization problems (such as (Schaffer 1985, Corne *et al.* 2000, Zitzler *et al.* 1999, 2001, Deb 2001, Deb *et al.* 2002)). A review by Coello (2002) lists and identifies the genetic algorithm based multi-objective optimization techniques.

3.2 Genetic Programming

Inspired from the genetic algorithm, Koza (1992) proposed the genetic programming approach in order to achieve a software program with a desired capability defined in terms of numerous input-output pairs. The approach is quite similar to the genetic algorithms; but this time, the genes inside the chromosomes are the building blocks (i.e. the functions, components or modules) of the sought computer program (Sette *et al.* 2001).

3.3 Differential Evolution

Extended from the genetic algorithm, the differential evolution is a recent metaheuristic originally proposed by Storn and Price (1997) for single-objective continuous problems. Again, the method relies on evolutionary operators such as crossover and mutation, in addition to the concept of so-called "differential weight". Despite its simplicity, the method has so far proven itself in numerous occasions with benchmark problems, and outperformed many other optimization algorithms. Later, many differential evolution variants for other purposes (i.e. for the solution of the continuous and multi-objective problems, etc.) have been proposed. Unlike other nature-inspired optimization methods, it is possible to guarantee the convergence of the trial-error procedure imposed by the algorithm.

3.4 Particle Swarm Optimization

Particle swarm optimization is a method proposed by Kennedy and Eberhart (1995) after getting inspired by the behaviours of the animal colonies/swarms. Similar to such swarms searching for the best place for nutrition in 3-dimensional space, this method relies on the motions of the swarm members (so-called "particles") searching for the global best in an N-dimensional continuous space. This time, the position of each particle is a candidate solution of the problem in hand. Each member of the swarm has:

- a cognitive behavior (i.e. having tendency to return positions related with good memories); as well as
- a social behavior (i.e. having tendency to go where the majority of the swarm members are located); in addition to
- an exploration capability (i.e. the tendency for random search throughout the domain).

The balance among these three tendencies is the key for the success and the power of the method. So far, the method has been successfully applied to various multidimensional continuous and discontinuous problems. In fact, the results of a similar analysis were recently reported by Poli (2008) in a review article (More detailed version of this review is also available on the web (Poli 2007)). The power of the method is its simplicity allowing implementation in almost every platform and every programming language as well as ease of parallelization.

Even though particle swarm optimization was originally proposed for single-objective continuous problems, later its discrete variants have also been published. Also, so far more than 30 versions of multi-objective particle swarm optimization extensions have been proposed, most of which have been reviewed by Reyes-Sierra and Coello (2006).

3.5 Ant Colony Optimization

Ant colony optimization is another algorithm originally proposed by Dorigo (1992) and later by Dorigo *et al.* (1996, 1997, 1999) for the solution of combinatorial problems such as the traveling salesman or the shortest path.

Dorigo was inspired from the behaviors of ants while transporting food to their nests. The algorithm depends on the following principles: Initially, ants have random movements; but upon finding food they lay down pheromone trails returning home. Other ants have tendency to follow these pheromones instead of keeping their random behavior. By the time, all pheromone trails start to evaporate and reduce their attractiveness. However, since pheromones over shorter paths are traced faster, and new pheromones are laid over the same path; new pheromone laying-out rate overcomes the evaporation rate. Due to this positive feedback mechanism, the popularity of shorter paths (i.e. pheromone density) increases in an accelerated manner. This is the key of success of the ant colony optimization for the solution of relevant problems.

Similarly, continuous and multi-objective variants of ant colony optimization have later been published.

3.6 Other Nature-Inspired Optimization Methods

There are some other recent algorithms with self-descriptive names such as the bees algorithm (Pham *et al.* 2006), artificial bee colony algorithm (Karaboga *et al.* 2007), saplings growing-up algorithm (Karci 2007), intelligent water drops algorithm (Shah-Hosseini 2009) etc., which are still waiting for further research and promotion.

4. APPLICATION OF METAHEURISTICS TO THE PORTFOLIO OPTIMIZATION PROBLEM: A LITERATURE REVIEW

As stated before, portfolio optimization problem is actually a constrained multi-objective optimization problem. But in the early decades (between 50s and early 90s), due to lack of powerful methods and computational power, the problem used to be handled with oversimplifying assumptions. With the diffusion of metaheuristics to all disciplines, researchers started to apply them to problems of their own branches. Eventually, portfolio optimization took its share. As of 2001, as pointed by Chen and Kuo (2001), there have been about 400 publications regarding the application of metaheuristics to problems in economy and finance. Certainly, publications related to portfolio optimization constituted the majority.

Early attempts were applications of metaheuristics to the single-objective portfolio optimization problem with no constraints. Dueck and Winker (1992) used a local-search based heuristic for the solution. Arnone *et al.* (1993) were the ones who first applied genetic algorithm to the portfolio selection problem. Later research can be considered in two main directions:

(i) incorporation of constraints in the problem model,

(ii) handling the problem as a multi-objective one.

Regarding the studies about incorporation of the constraints in the problem: Jobst *et al.* (2001) and Fieldsend *et al.* (2004) discussed the computational aspects in presence of the discrete asset choice constraints. Chang *et al.* (2000) applied tabu search, simulated annealing and genetic algorithm to the portfolio optimization problem considering the cardinality constraint; afterwards, Schaerf (2002) and Kellerer *et al.* (2003) applied local search algorithms; Streichert *et al.* (2003) and Diosan (2005) applied various evolutionary algorithms for the same purpose.

One of the very early multi-objective solution approaches in portfolio optimization problem was by Lin *et al.* (2001), who applied genetic algorithm for this purpose. Crama and Schyns (2003) disussed how to apply simulated annealing in complex portfolio optimization problems. Ong *et al.* (2005) applied a multi-objective evolutionary algorithm, whereas Armananzas and Lozano (2005) applied multi-objective greedy-search, simulated annealing and ant colony optimization by considering the portfolio optimization problem as a tri-objective one. Doerner *et al.* (2001) applied ant colony optimization; Subbu *et al.* (2005), Diosan (2005) and Chiam *et al.* (2008) applied various evolutionary algorithms; Kendall and Su applied particle swarm optimization; Yang (2006) applied the genetic algorithm. Application of differential evolution to the field is brand new. As of today, Ardia *et al.* (2010) and Krink *et al.* (2010) are the ones who have so far applied differential evolution to the portfolio optimization problem.

For deeper and broader surveys of the literature, interested readers can take a look at Schlottmann and Seese (2004) or Tapia and Coello (2007) the on application of multi-objective evolutionary algorithms in economics and finance in general.

Application of metaheuristics, or more specifically nature-inspired methods to similar problems such as index fund management, credit portfolio construction, etc. is also possible. Orito *et al.* (2003), Kyong *et al.* (2005), Oh *et al.* (2005, 2006) used genetic algorithms for the index fund management problem.

Another issue in portfolio management is the cost of transactions, which is usually neglected during the modeling of the problems. It is possible to incorporate this factor while applying metaheuristics. For example, Chen and Zhang (2010) applied a particle swarm optimization variant to the portfolio optimization problem considering the transaction costs.

Meanwhile, genetic programming also found application in finance. Potvin *et al.* (2004) applied genetic programming for generating trading rules in stock markets; more recently, Etemadi *et al.* (2009) used it for bankruptcy prediction, meanwhile Chen *et al.* (2010) used a time adaptive version of the technique to portfolio optimization.

5. CONCLUSIONS

Metaheuristics, more specifically nature-inspired optimization algorithms constitute powerful means for the solution of existing problems in economy and finance. The main factors promoting the usage of such algorithms can be summarized as follows:

- The algorithms make no assumptions (or require no *a priori* information) about the objective function.
- They do not require the objective function to be continuous or differentiable.
- They can handle complicated models with constraints.
- All of them have variants for continuous and combinatorial problems.
- All of them have variants extended for multi-objective problems.
- Almost all of them support parallelization, which yields the solution of very large-scale problems.

Eventually, the literature is full of a plethora of publications about successful applications of such algorithms to the problems, which could not have been considered and handled previously with conventional approaches. By observing the rate of increase in such publications, it can be easily forecasted that more and more applications will occur in the near future.

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