Pressure in MHD/Brinkman flow past a stretching sheet

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Abstract: This paper deals with the pressure in steady two-dimensional/axisymmetric MHD/Brinkman flow of an incompressible viscous electrically conducting fluid over a flat stretching sheet. Stretching rate of two-dimensional case is assumed as double the stretching rate of axisymmetric case. A recently proposed approximate analytic technique (Kumaran et al. (2009)) was used to recover an exact solution of the two-dimensional case, to develop an approximate analytical solutions of the axisymmetric case and were discussed by Kumaran and Tamizharasi (2010). In this paper, the pressure distribution is studied in detail. The pressure distribution of MHD and porous cases are plotted and compared. Also two-dimensional and axisymmetric cases are compared by means of a unified scale.

Keywords: steady viscous two-dimensional/axisymmetric flow, MHD, porous medium, stretching sheet, pressure distribution

1. INTRODUCTION

Flow past a stretching sheet has several important engineering applications in the polymer processing unit of a chemical plant and for the metal working process in metallurgy. Flow of an incompressible viscous fluid over a stretching sheet has an important bearing on several technological processes, like in the extrusion of a polymer in a melt-spinning process etc. Further the study of magnetohydrodynamic(MHD) flow of an electrically conducting fluid caused by the deformation of the walls of the vessel containing this fluid is of considerable interest in modern metallurgical and metal-working processes. An exact similarity solution of the MHD boundary layer equations for the steady two-dimensional flow of an electrically conducting incompressible fluid due to the stretching of a plane elastic surface in the presence of a uniform transverse magnetic field is presented by Pavlov (1974). The temperature distribution in this flow when a uniform suction is applied at the stretching surface is found by Chakrabarti and Gupta (1979).

Andersson (1995) demonstrated that the similarity solution obtained by Pavlov (1974) is not only a solution of the boundary layer equation but also represents an exact solution of the Navier-Stokes equations for magnetohydrodynamic flow. Ariel (1996) demonstrated that judiciously produced approximate solutions of physical problems are of high interest as they serve the practical purposes. Applying the idea of stretching the variables of the flow concerned, he obtained a useful solution for the problem of flow near a rotating disc. The MHD flow past a stretching surface of a viscoelastic fluid is studied by Andersson (1992), Ariel (1994), Dandapat et al. (1998), Sujit et al. (2003), Abel and Nandeppanavar (2009), power-law fluid by Andersson et al. (1992), micro-polar fluid by Eldabe et al. (2005), second grade fluid by Sahoo (2010), upper convected Maxwell fluid by Raftari and Yildirim (2010) and unsteady three-dimensional fluid flow by Mehmood et al. (2008) and Takhar et al. (2001) subject to various physical characteristics. The magnetohydrodynamic flow over a stretching sheet was further studied by Vajravelu (1986), Takhar et al. (1987), Takhar et al. (1989), Kumari et al. (1990), Vajravelu and Rollins (1992), Pop and Na (1998), Chakraborty and Mazumdar (2000), Liao (2003), Amkadni et al. (2008), Ishak et al. (2008), Kumaran et al. (2009a) and Fang et al. (2009).

The Darcy, Brinkman and Forchhrimer equations describing the flow in a porous medium have been extensively studied in Stranghan (1993). The flow along a stretching permeable surface in Darcy-Brinkman porous medium has been investigated by Pantokratoras (2009). Various characteristics of fluid immersed in a porous medium over astretching sheet is also studied by Chamkha (1998) and Liu (2006).

Though lot of work has been done on MHD/Brinkman flows past a stretching sheet independently, to the best of our knowledge a more detailed study of pressure distribution of two-dimensional and axisymmetric cases has not been done so far. Hence in the present paper, the steady two-dimensional/axisymmetric flow of an incompressible viscous MHD/Brinkman flow past a stretching sheet is revisited. Also pressure field is analysed and compared between the MHD and porous medium cases.

2. MATHEMATICAL FORMULATION: TWO-DIMENSIONAL CASE

Consider a two-dimensional steady flow of an incompressible viscous electrically conducting fluid impinging vertically downwards on a horizontal deformable sheet, stretched in its own plane $\bar{z} = 0$, from $\bar{x}$=0, $\bar{z}$=0 where $\bar{x}$, $\bar{z}$ are cartesian co-ordinates along the horizontal and vertical directions. Velocity field in cartesian form is given by $\bar{V} = (\bar{u}, 0, \bar{v})$ where $\bar{u}$, $\bar{v}$ are the velocity components along $\bar{x}$, $\bar{z}$ directions repectively. The stretching velocity is $U_0 = 2k_0\bar{x}$, along $\bar{x}$ direction where $2k_0$ ($k_0 > 0$) is the constant stretching rate. K is the permeability of the porous medium, B is a constant applied magnetic field and ν, ρ, σ are the kinematic viscosity, density, electrical conductivity of the fluid respectively. Assuming $K \to \infty$ when $B \neq 0$, $B = 0$ when K is finite and $\bar{p}(\bar{x}, \bar{z})$ as the pressure distribution, the flow is governed by the following equations:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{z}} = 0 \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{z}} = \nu(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{z}^2}) - \frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{x}} - D_1\bar{u} \tag{2}$$

$$\bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{z}} = \nu(\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{z}^2}) - \frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{z}} - D_2\bar{v} \tag{3}$$

with the boundary conditions,

$$\bar{u} = U_0, \quad \bar{v} = 0 \quad at \quad \bar{z} = 0, \tag{4}$$

$$\bar{u} \to 0 \quad as \quad \bar{z} \to \infty. \tag{5}$$

where $D_1 = \frac{\sigma B^2}{\rho}$, in case of MHD flow and $\frac{\nu}{K}$, in case of porous medium. $D_2 = 0$, in the case of MHD flow and $\frac{\nu}{K}$, in case of porous medium. The magnetic field with strength B is applied in the vertical direction and the induced magnetic field is neglected. Using the velocity scale $C = \sqrt{2\nu k_0}$, the dimensionless variables and the non-dimensional parameters are defined as,

$$(\xi, \eta) = \frac{C}{\nu}(\bar{x}, \bar{z}), (u, v) = \frac{1}{C}(\bar{u}, \bar{v}),$$
$$\psi = \frac{\bar{\psi}}{\nu}, p = \frac{\bar{p}}{\rho C^2}, \beta = \frac{\nu D_1}{C^2}, \tag{6}$$

where $\bar{\psi}(\bar{x}, \bar{z})$ is the stream function which is given by,

$$\bar{u}(\bar{x}, \bar{z}) = \frac{\partial \bar{\psi}}{\partial \bar{z}}, \quad \bar{v}(\bar{x}, \bar{z}) = -\frac{\partial \bar{\psi}}{\partial \bar{x}}. \tag{7}$$

Boundary conditions (4) and (5) suggest ψ in the dimensionless form as,

$$\psi(\xi, \eta) = \xi g(\eta). \tag{8}$$

Equations (4) and (5) become,

$$g(0) = 0, \qquad g'(0) = 1, \qquad g'(\infty) = 0, \tag{9}$$

where primes denote the differentiation with respect to η. Using equations (6)-(8), equations (1) -(3) become,

$$g''' + gg'' - g'^2 - \beta g' = 0, \tag{10}$$

For MHD flow, the dimensionless excess pressure with respect to the dimensionless stagnation point pressure $p(0, 0)$ takes the form,

$$\Delta p = p(\xi, \eta) - p(0, 0) = 1 - g' - \frac{g^2}{2}, \tag{11}$$

whereas for the porous medium case, it is of the form,

$$\Delta p = p(\xi, \eta) - p(0, 0) = 1 - g' - \frac{g^2}{2} + \beta\int_0^\eta g d\eta. \tag{12}$$

In both the cases, as Δp is independent of ξ, the dimensionless excess centreline pressure (Δp at $\xi = 0$) is given by,

$$\Delta p_c = p(0, \eta) - p(0, 0)) = \Delta p. \tag{13}$$

2.1 Analytic solution technique

Consider the following scaled variables,

$$x = \frac{\eta}{\sqrt{\epsilon}}, \qquad F(x) = \frac{g(\eta)}{\sqrt{\epsilon}}, \quad \epsilon = \frac{1}{a + \beta}. \tag{14}$$

where a, an artificial non–negative parameter a, to be found later. Using the above variables in (10) and (9), we get,

$$F''' - F' + \epsilon(FF'' - F'^2 + aF') = 0, \tag{15}$$

$$F(0) = 0, \qquad F'(0) = 1, \qquad F'(\infty) = 0. \tag{16}$$

Expanding $F(x)$ in powers of ϵ as,

$$F(x) = F_0(x) + \epsilon F_1(x) + \epsilon^2 F_2(x) + ... \tag{17}$$

Using (17) in (15)-(16) and collecting co-efficients of powers of ϵ, we get the zeroth order boundary value problem as,

$$F_0''' - F_0' = 0, \tag{18}$$

$$F_0(0) = 0, \quad F_0'(0) = 1, \quad F_0'(\infty) = 0. \tag{19}$$

Note that $F_k(x)$, $k \geq 0$ are functions of x. Since $F(x)$ is a series in powers of ϵ, we consider only n^{th} order approximate solutions in the form,

$$g(\eta) \approx g_n(\eta) = \sqrt{\epsilon}(\sum_{k=0}^{n} \epsilon^k F_k(x)). \tag{20}$$

Also, the total residual of (10) with respect to $g_n(\eta)$ is defined as,

$$R(n, a, \beta) = \sqrt{\int_0^\infty (g_n''' + g_n g_n'' - g_n'^2 - \beta g_n')^2 d\eta} \tag{21}$$

It is also possible to get the regular perturbation solution of (10) and (9), if a is chosen as zero in (14).

2.2 Solution for two-dimensional case

For the two-dimensional case, the zeroth order solution yields the values of the artificial non-negative parameter a and the corresponding redidual error R as $a_0 = 1$ (obtained by minimizing the equation (21)) and $R_0 = 0$ for $\beta = 0$. As a result, the zeroth order solution itself is an exact analytical solution of (10) and (9), which holds good for both MHD/Brinkman flow and is given by

$$g(\eta) \equiv g_0(\eta) = \frac{1 - e^{-\eta\sqrt{1+\beta}}}{\sqrt{1 + \beta}}. \tag{22}$$

This solution is also reported in Andersson (1992) and Dandapat and Gupta (1989). For the two-dimensional

MHD flow, the dimensionless excess pressure distribution is given in the form,

$$\Delta p = \frac{1}{2(1+\beta)}[(1 - e^{-\eta\sqrt{1+\beta}})(1 + 2\beta + e^{-\eta\sqrt{1+\beta}})]. \quad (23)$$

And the dimensionless excess pressure distribution of the two-dimensional porous medium case takes the form,

$$\Delta p = \frac{1}{2(1+\beta)}[1 - e^{-2\eta\sqrt{1+\beta}} + 2\beta\eta\sqrt{1+\beta}]. \quad (24)$$

3. AXISYMMETRIC CASE

Consider a viscous incompressible MHD/Brinkman flow past a horizontal deformable sheet stretching radially in its own plane with radial speed $U_0 = k_0\bar{x}$, where $k_0(>0)$ is the stretching rate and $\bar{x}$ is the radial co-ordinate. Taking vertical co-ordinate as $\bar{z}$ with the assumption that the sheet is on $\bar{z} = 0$. The velocity field in cylindrical co-ordinates is given by $\bar{V} = (\bar{u}, 0, \bar{v})$ where $\bar{u}$, $\bar{v}$ are the velocity components along $\bar{x}$, $\bar{z}$ directions respectively with $\bar{p}(\bar{x}, \bar{z})$ is the pressure field, ν, ρ, K and σ are as in section 2, the Navier-Stokes equations of rotational symmetry can be written as,

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{z}} = \frac{-1}{\rho}\frac{\partial \bar{p}}{\partial \bar{x}} + \nu\left[\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{1}{\bar{x}}\frac{\partial \bar{u}}{\partial \bar{x}} - \frac{\bar{u}}{\bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{z}^2}\right] - D_1\bar{u}, \quad (25)$$

$$\bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{z}} = \frac{-1}{\rho}\frac{\partial \bar{p}}{\partial \bar{z}} + \nu\left[\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \frac{1}{\bar{x}}\frac{\partial \bar{v}}{\partial \bar{x}} + \frac{\partial^2 \bar{v}}{\partial \bar{z}^2}\right] - D_2\bar{v}, \quad (26)$$

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\bar{u}}{\bar{x}} + \frac{\partial \bar{v}}{\partial \bar{z}} = 0, \quad (27)$$

where D_1 and D_2 are as defined in two-dimensional case. The boundary conditions take the form,

$$\bar{u} = U_0, \quad \bar{v} = 0 \quad at \quad \bar{z} = 0, \quad (28)$$

$$\bar{u} = 0 \quad as \quad \bar{z} \to \infty. \quad (29)$$

The non-dimensional form of $\bar{\psi}$ is,

$$\psi = \frac{C\bar{\psi}}{\nu^2}. \quad (30)$$

The velocity scale C, other dimensionless variables and the non-dimensional parameters are as defined in section 2. The stream function $\bar{\psi}(\bar{x}, \bar{z})$ is given by,

$$\bar{u}(\bar{x}, \bar{z}) = \frac{1}{\bar{x}}\frac{\partial \bar{\psi}}{\partial \bar{z}}, \quad \bar{v}(\bar{x}, \bar{y}) = \frac{-1}{\bar{x}}\frac{\partial \bar{\psi}}{\partial \bar{x}}. \quad (31)$$

Boundary conditions (28) and (29) suggest ψ in the dimensionless form as,

$$\psi(\xi, \eta) = \frac{\xi^2}{2}g(\eta). \quad (32)$$

Then using equations (6) except for ψ, (29)-(31), the equations
(25) -(29) become,

$$g''' + gg'' - \frac{1}{2}g'^2 - \beta g' = 0, \quad (33)$$

$$g(0) = 0, \quad g'(0) = 1, \quad g'(\infty) = 0. \quad (34)$$

The axisymmetric pressure distribution is the same as defined for the two-dimensional case except that $g(\eta)$ is given by (33)-(34).

3.1 Solution for axisymmetric case

Using the scaled variables defined in equation (14), equations (33)-(34) become

$$F''' - F' + \epsilon(FF'' - \frac{1}{2}F'^2 + aF') = 0, \quad (35)$$

$$F(0) = 0, \quad F'(0) = 1, \quad F'(\infty) = 0. \quad (36)$$

Again using (17) in equations (35)and (36) and collecting co-efficients of powers of ϵ, we get equations (18)-(19) as the zeroth order boundary value problem and the approximate solutions are also considered in the form of equation (20). And the total residual of (33) with respect to $g_n(\eta)$ is defined as,

$$R(n, a, \beta) = \sqrt{\int_0^\infty (g_n''' + g_n g_n'' - \frac{1}{2}g_n'^2 - \beta g_n')^2 d\eta} \quad (37)$$

For this axisymmetric case, the obtained zeroth order a and R values are $a_0 = 0.68750$ and $R_0 = 0.12309$.
As finding exact analytical solution of (33) with respect to (34) is not possible by us, we obtain an approximate analytical solution of the same upto eight order for the axisymmetric (Brinkman) case and is briefly explained in the paper communicated by Kumaran and Tamizharasi (2010) using an analytic technique proposed by Kumaran et al. (2009). The dimensionless excess pressure distribution of the axisymmteric (MHD/Brinkman) case is found using the eighth order approximation for $g(\eta)$ given by (33) and (34).

4. DISCUSSION

Table 1 shows the values of the ratio (between two-dimensional and axisymmetric) of excess pressures as $\eta \to \infty$ of MHD and porous cases for various values of β. The observations made are:
(i) When $\beta = 0$, for both MHD and porous medium cases, far away from the sheet, the excess pressure of two-dimensional flow is ≈ 1.15 times the axisymmetric excess pressure.
(ii) As β increases, the ratio of excess pressures as $\eta \to \infty$ of MHD flow gradually decreases and approaches 1, when $\beta \to \infty$.
(iii) For the porous medium case, as $\eta \to \infty$, the ratio of excess pressures decrease, attains local minimum excess pressure ratio 0.94116 at $\beta = 0.006$, then behaves in an increasing fashion and thereby reaches 1, when $\beta \to \infty$.
Figs. 1 and 2 show the profiles of excess pressure of the two-dimensional flow of MHD and porous medium cases for various values of β. The remarks made are:
(i) In both MHD and porous medium cases, near the sheet, as β and η increases, excess pressure also increases.
(ii) For the MHD flow, irrespective of the values of β, excess pressure becomes constant for large η.
(iii) In porous medium case, for large η, excess pressure

Table 1. Values of ratio of excess pressures $\frac{\Delta p_0(\infty)}{\Delta p_8(\infty)}$ for the MHD and porous medium cases ($\Delta p_0(\infty)$: two-dimensional, $\Delta p_8(\infty)$: axisymmetric)

β	MHD	porous
0	1.14892	1.14892
1	1.02684	0.96295
5	1.00250	0.98654
10	1.00072	0.99255
50	1.00003	0.99837
100	1.00001	0.99918

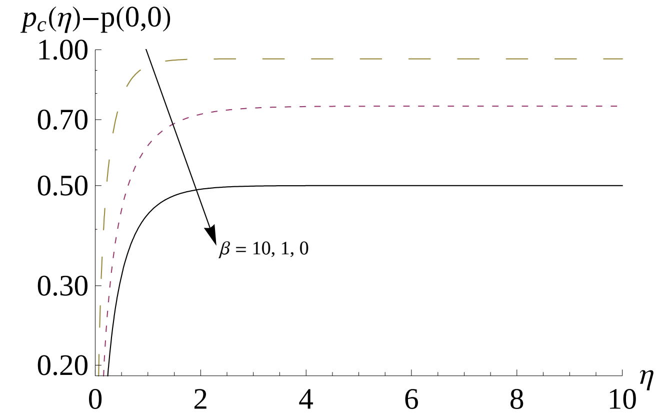

Fig. 1. nmhdp: Profiles of Δp Vs η: MHD two-dimensional case(0^{th} order)

becomes constant for $\beta = 0$. But for $\beta > 0$, as η increases, pressure is completely in an increasing manner and it goes up to infinity. It is more pronounced with β.

Figs. 3 and 4 display the profiles of ratio of excess centerline pressure of two-dimensional and axisymmetric flows of the MHD and porous medium cases for various values of β. Fig. 4 is also discussed in Kumaran and Tamizharasi (2010). Some observations made from the figures are:

(i) $\Delta(\Delta p_c)$ decreases for increasing β near the sheet.

(ii) Near the sheet: $\Delta(\Delta p_c)$ is uniform to some extent of η, then keeps reducing and after some specific value of η, it later becomes a constant in the potential flow, for increasing η and for all β. It is seen that two-dimensional (in both MHD and porous medium cases) Δp_c is slightly greater than the axisymmetric (in both MHD and porous medium cases) Δp_c, as two-dimensional Δp_c is

$\approx$ 1.21 times the axisymmetric Δp_c, in case of β=0.

$\approx$ 1.09 times the axisymmetric Δp_c, in case of β=1.

$\approx$ 1.02 times the axisymmetric Δp_c, in case of β=10.

(iii) Away from the sheet: It is interesting to note the existence of cross over of $\beta = 1$ profile with that of $\beta = 10$ at some particular η, in the porous medium case. Due to that cross over, $\Delta(\Delta p_c)$ corresponding to $\beta = 1$ is dominated by $\beta = 10$ case. It is also confirmed by table 1. It is instructive to note that all these results correspond to stretching rate of two-dimensional case is double that of the stretching rate of axisymmetric case.

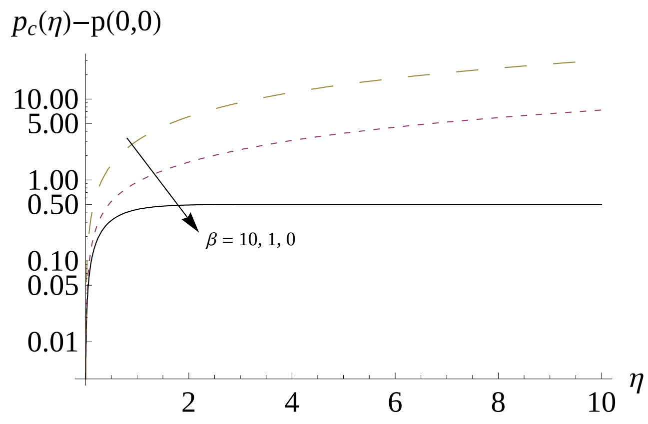

Fig. 2. nsd: Profiles of Δp Vs η: porous medium two-dimensional case(0^{th} order)

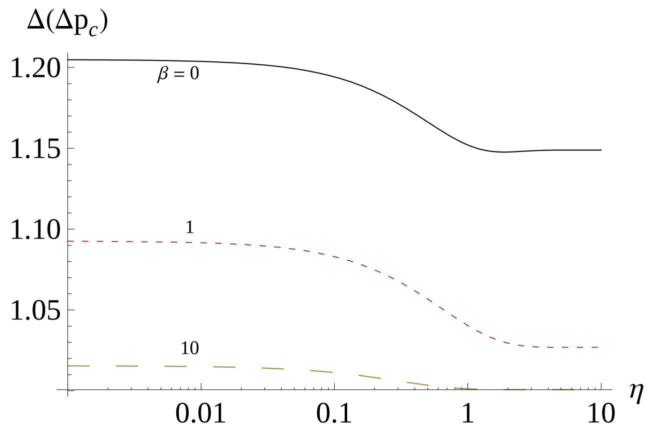

Fig. 3. mhdp1: Profiles of ratio of Δp's of two-dimensional and axisymmetric cases (MHD case)

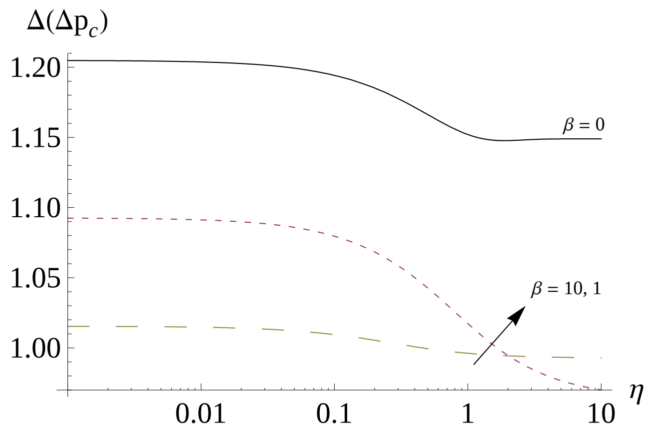

Fig. 4. sc: Profiles of ratio of Δp's of two-dimensional and axisymmetric cases (porous medium case)

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