Desirable role in an international market with uncertainty

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Abstract: This paper examines optimal trade in a market with demand uncertainty, in a duopoly in which a home firm competes with a foreign firm. The home government chooses an import tariff to maximize the revenue. Each firm is a Cournot competitor or a Stackelberg leader. The uncertainty is resolved between the decisions made by the home government and by the firms. We compute the maximum-revenue tariff, the quantities, the prices and the profits in each role of the model. We compare the results obtained in the three different ways of moving on the decision make.

Keywords: Game Theory, optimization problems, uncertainty, Cournot model, leadership.

1. INTRODUCTION

It is well known that the outcome of a duopoly market varies when competition takes different forms such as Cournot or Stackelberg. Most of the economic literature assumes Cournot competition with simultaneous play as the natural order of moves in a quantity-setting game. However, recent advances in game theory argue that the assumed order of play should also be consistent with the players preferences over the time of actions. There are several comparative studies of these market structures. Notice that, in the typical models, compared to Cournot competition the Stackelberg leader can never be worse off: The leader can choose the Cournot quantity and then for the follower it is optimal to produce the same amount and we have Cournot outcome; So, leader either chooses Cournot quantity or if it chooses a different level of production it must be better off otherwise it wouldn't do that. Here, we examines optimal trade in a market with demand uncertainty, in a duopoly in which a home firm competes with a foreign firm. Furthermore, we also assume that in that international market the home government imposes a tariff on the imported goods. In this case, we will see that the results can be different than the ones refereed above.

Tariff revenue may be an important source of government revenue for developing countries that do not have an efficient tax system. So, the government may use the maximum-revenue tariff. Brander and Spencer (1984) have shown that a tariff has a profit-shifting effect in addition to its effect on tariff revenue. Larue and Gervais (2002) studied the effect of maximum-revenue tariff in a Cournot duopoly. Ferreira and Ferreira (2009) examined the maximum-revenue tariff under international Bertrand competition with differentiated products when rivals' production costs are unknown. Clarke and Collie (2006) studied a similar question, when there is no uncertainty on the production costs. The propose of this paper is to study the maximum-revenue tariff under international quantity competition with demand uncertainty, with different possible timings of decisions.

We consider a two-country, two-good model where a domestic and a foreign good are produced by a home and a foreign monopolist, respectively. Since we assume that the two countries are perfectly symmetric, it is sufficient to describe only the domestic economy. We should mention that issues related to those of this paper have been studied by Ferreira et al. (2007a), Ferreira et al. (2007b), Ferreira and Pinto (2008) and Spulber (1995).

2. THE BENCHMARK MODEL

There are two countries, home and foreign. Each country has one firm, firm F_1 (home firm) and firm F_2 (foreign firm), that produces homogeneous goods. Consider the home market, where the two firms compete in quantities (see Tirole (1994)). We consider that the domestic government imposes an import tariff t per unit of imports from the foreign firm.

The inverse demand function is given by

$$p = A - q_i - q_j,$$

where $i, j \in \{1, 2\}$ with $i \neq j$ and q_i stands for quantity. In this section, we assume that the intercept A is commonly known since the begin of the game.

The model consists in the following two-stage game:

- In the first stage, the domestic government chooses the import tariff t per unit of imports from the foreign firm.
- In the second stage, both firms choose output levels.

Firms' profits, π_1 and π_2, are given by

$$\pi_1 = (A - q_1 - q_2)q_1,$$
$$\pi_2 = (A - q_1 - q_2 - t)q_2.$$

2.1 Simultaneous decision

In this section, we suppose that, in the second stage of the game, both home and foreign firms play a Cournot-type game, i.e., each firm F_i independently chooses q_i. Let the superscript C denote the equilibrium outcome of the Cournot-type game.

We determine the subgame perfect Nash equilibrium by backwards induction. Suppose that the domestic government has chosen the import tariff t per unit of imports in the first stage.

Maximizing simultaneously both firms' profits

$$\pi_1 = (A - q_1 - q_2)q_1$$

and

$$\pi_2 = (A - q_1 - q_2 - t)q_2,$$

we get the following output levels:

$$q_1 = \frac{A + t}{3}$$

and

$$q_2 = \frac{A - 2t}{3}.$$

Now, we can use the above results to derive the maximum-revenue tariff. The maximum-revenue tariff is the tariff rate that maximizes the tariff revenue collected by the government in the home country:

$$R = t\frac{A - 2t}{3}.$$

Theorem 1. In the Cournot-type game, the maximum-revenue tariff is given by

$$t^C = \frac{A}{4}. \quad (1)$$

So, we get the following result.

Theorem 2. In the case of Cournot competition, the output levels at equilibrium are given by

$$q_1^C = \frac{5A}{12}$$

and

$$q_2^C = \frac{A}{6}.$$

Thus, the aggregate quantity in the market is given by

$$Q^C = \frac{7A}{12}$$

and the price is given by

$$p^C = \frac{5A}{12}.$$

The following results are also obtained straightforwardly.

Theorem 3. In the case of Cournot competition, home firm's profit is given by

$$\pi_1^C = \frac{25A^2}{144},$$

and foreign firm's profit is given by

$$\pi_2^C = \frac{A^2}{36}.$$

Corollary 4. In the case of Cournot competition, home firm profits more than foreign firm.

2.2 Home firm is the leader

In this section, we suppose that, in the second stage of the game, the home firm is the leader. Home firm F_1 chooses q_1, and foreign firm F_2 chooses q_2 after observing q_1. Let the superscript L denote the equilibrium outcome of the game where the home firm F_1 is the leader.

We determine the subgame perfect Nash equilibrium by backwards induction. Suppose that the domestic government has chosen the import tariff t per unit of imports in the first stage. Also, suppose that, the leader home firm F_1 produces q_1. Then, maximizing foreign firm profit's

$$\pi_2 = (A - q_1 - q_2 - t)q_2,$$

we get

$$q_2 = \frac{A - t - q_1}{2}. \quad (2)$$

Now, maximizing home firm profit's

$$\pi_1 = (A - q_1 - q_2)q_1,$$

knowing the above quantity q_2, we get

$$q_1 = \frac{A + t}{2}. \quad (3)$$

Putting equation (10) into equation (9), we get

$$q_2 = \frac{A - 3t}{4}.$$

Now, we can use the above results to derive the maximum-revenue tariff. The maximum-revenue tariff is the tariff rate that maximizes the tariff revenue collected by the government in the home country. From the above results, the tariff revenue is

$$R = t\frac{A - 3t}{4},$$

which leads us to the following result.

Theorem 5. In the case of Stackelberg competition, with the home firm as the leader, the maximum-revenue tariff is given by

$$t^L = \frac{A}{6}. \quad (4)$$

So, the output levels at equilibrium are as follows.

Theorem 6. In the case of Stackelberg competition, with the home firm as the leader, the home firm produces

$$q_1^L = \frac{7A}{12},$$

and the foreign firm produces

$$q_2^L = \frac{A}{8}.$$

Thus, the aggregate quantity in the market is given by

$$Q^L = \frac{17A}{24}$$

and the price is given by

$$p^L = \frac{7A}{24}.$$

The following results are also obtained straightforwardly.

Theorem 7. In the case of Stackelberg competition, with the home firm as the leader, home firm's profit is given by

$$\pi_1^L = \frac{49A^2}{288},$$

and foreign firm's profit is given by

$$\pi_2^L = \frac{A^2}{64}.$$

2.3 Home firm is the follower

In this section, we suppose that, in the second stage of the game, the home firm is the follower. Foreign firm F_2 chooses q_2, and home firm F_1 chooses q_1 after observing q_2. Let the superscript F denote the equilibrium outcome of the game where the home firm F_1 is the follower.

We determine the subgame perfect Nash equilibrium by backwards induction. Suppose that the domestic government has chosen the import tariff t per unit of imports in the first stage. Also, suppose that, the leader foreign firm F_2 produces q_2. Then, maximizing home firm profit's

$$\pi_1 = (A - q_1 - q_2)q_1,$$

we get

$$q_1 = \frac{A - q_2}{2}. \tag{5}$$

Now, maximizing foreign firm profit's

$$\pi_2 = (A - q_1 - q_2 - t)q_2,$$

knowing the above quantity q_1, we get

$$q_2 = \frac{A - 2t}{2}. \tag{6}$$

Putting equation (13) into equation (12), we get

$$q_1 = \frac{A + 2t}{4}.$$

Now, we can use the above results to derive the maximum-revenue tariff. The maximum-revenue tariff is the tariff rate that maximizes the tariff revenue collected by the government in the home country. From the above results, the tariff revenue is

$$R = t\frac{A - 2t}{4},$$

which leads us to the following result.

Theorem 8. In the case of Stackelberg competition, with the home firm as a follower, the maximum-revenue tariff is given by

$$t^F = \frac{A}{4}. \tag{7}$$

So, the output levels at equilibrium are as follows.

Theorem 9. In the case of Stackelberg competition, with the home firm as a follower, the home firm produces

$$q_1^F = \frac{3A}{8}$$

and the foreign firm produces

$$q_2^F = \frac{A}{4}.$$

Thus, the aggregate quantity in the market is given by

$$Q^F = \frac{5A}{8}$$

and the price is given by

$$p^F = \frac{3A}{8}.$$

The following result is also obtained straightforwardly.

Theorem 10. In the case of Stackelberg competition, with the home firm as a follower, home firm's profit is given by

$$\pi_1^F = \left(\frac{3A}{8}\right)^2$$

and foreign firm's profit is given by

$$\pi_2^F = \frac{A^2}{32}.$$

Corollary 11. In the case of Stackelberg competition, with the home firm as a follower, the home firm profits more than the foreign leader firm.

3. THE MODEL WITH DEMAND UNCERTAINTY

In this section, we consider that the inverse demand function is given by

$$p = A - q_i - q_j,$$

where the intercept A is ex-ante unobservable, although its prior cumulative function $F(a)$ is commonly known to domestic government and both firms, with strictly positive finite mean $E(A)$ and variance $V(A)$. We assume that the exact realization of this intercept A becomes observable after the decision on the import tariff t fixed by the domestic government, and before both firms decide their output levels.

The model consists in the following two-stage game:

- In the first stage, the domestic government chooses the import tariff t per unit of imports from the foreign firm, without knowing the demand realization.
- In the second stage, both firms choose output levels, knowing the exact realization a of the demand,

Firms' profits, π_1 and π_2, are given by

$$\pi_1 = (a - q_1 - q_2)q_1,$$
$$\pi_2 = (a - q_1 - q_2 - t)q_2.$$

3.1 *Simultaneous decision*

In this section, we suppose that, in the second stage of the game, both home and foreign firms play a Cournot-type game, i.e., each firm F_i independently chooses q_i. Let the superscript C denote the equilibrium outcome of the Cournot-type game.

We determine the subgame perfect Nash equilibrium by backwards induction. Suppose that the domestic government has chosen the import tariff t per unit of imports in the first stage.

Maximizing simultaneously both firms profits

$$\pi_1 = (a - q_1 - q_2)q_1$$

and

$$\pi_2 = (a - q_1 - q_2 - t)q_2,$$

we get the following output levels:

$$q_1 = \frac{a+t}{3}$$

and

$$q_2 = \frac{a-2t}{3}.$$

Now, we can use the above results to derive the maximum-revenue tariff. The maximum-revenue tariff is the tariff rate that maximizes the tariff revenue collected by the government in the home country. Since the government does not know the exact demand, it will use the expected demand to compute that tariff. The tariff is t per unit of imports and the expected demand is $E(A)$; so, expected tariff revenue is

$$E(R) = t\frac{E(A)-2t}{3}.$$

Theorem 12. The maximum-revenue tariff is given by

$$t^C = \frac{E(A)}{4}. \tag{8}$$

So, we get the following result.

Theorem 13. In the case of Cournot competition, the output levels at equilibrium are given by

$$q_1^C = \frac{4a+E(A)}{12}$$

and

$$q_2^C = \frac{2a-E(A)}{6}.$$

Thus, the aggregate quantity in the market is given by

$$Q^C = \frac{8a-E(A)}{12}$$

and the price is given by

$$p^C = \frac{4a+E(A)}{12}.$$

The following result is also obtained straightforwardly.

Theorem 14. Expected ex-ante profits of the two firms are given by

$$E(\pi_1^C) = \left(\frac{5E(A)}{12}\right)^2 + \frac{V(A)}{9}$$

and

$$E(\pi_2^C) = \left(\frac{E(A)}{6}\right)^2 + \frac{V(A)}{9}.$$

3.2 *Home firm is the leader*

In this section, we suppose that, in the second stage of the game, the home firm is the leader. Home firm F_1 chooses q_1, and foreign firm F_2 chooses q_2 after observing q_1. Let the superscript L denote the equilibrium outcome of the game where the home firm F_1 is the leader.

We determine the subgame perfect Nash equilibrium by backwards induction. Suppose that the domestic government has chosen the import tariff t per unit of imports in the first stage. Also, suppose that, the leader home firm F_1 produces q_1. Then, maximizing foreign firm profit

$$\pi_2 = (a - q_1 - q_2 - t)q_2,$$

we get

$$q_2 = \frac{a-t-q_1}{2}. \tag{9}$$

Now, maximizing home firm profit

$$\pi_1 = (a - q_1 - q_2)q_1,$$

knowing the above quantity q_2, we get

$$q_1 = \frac{a+t}{2}. \tag{10}$$

Putting equation (10) into equation (9), we get

$$q_2 = \frac{a-3t}{4}.$$

Now, we can use the above results to derive the maximum-revenue tariff. The maximum-revenue tariff is the tariff rate that maximizes the expected tariff revenue collected by the government in the home country. From the above results, the expected tariff revenue is

$$E(R) = t\frac{E(A)-3t}{4},$$

which leads us to the following result.

Theorem 15. In the case of Stackelberg competition, with the home firm as a leader, the maximum-revenue tariff is given by

$$t^L = \frac{E(A)}{6}. \tag{11}$$

So, the output levels at equilibrium are as follows.

Theorem 16. In the case of Stackelberg competition, with the home firm as a leader, the home firm produces

$$q_1^L = \frac{6a+E(A)}{12}$$

and the foreign firm produces

$$q_2^L = \frac{2a-E(A)}{8}.$$

Thus, the aggregate quantity in the market is given by

$$Q^L = \frac{18a - E(A)}{24}$$

and the price is given by

$$p^L = \frac{6a + E(A)}{24}.$$

The following result is also obtained straightforwardly.

Theorem 17. In the case of Stackelberg competition, with the home firm as the leader, home firm's ex-ante expected profit is given by

$$E(\pi_1^L) = \frac{49(E(A))^2}{288} + \frac{36V(A)}{288}$$

and foreign firm's ex-ante expected profit is given by

$$E(\pi_2^L) = \frac{(E(A))^2}{64} + \frac{V(A)}{16}.$$

3.3 Home firm is the follower

In this section, we suppose that, in the second stage of the game, the home firm is the follower. Foreign firm F_2 chooses q_2, and home firm F_1 chooses q_1 after observing q_2. Let the superscript F denote the equilibrium outcome of the game where the home firm F_1 is the follower.

We determine the subgame perfect Nash equilibrium by backwards induction. Suppose that the domestic government has chosen the import tariff t per unit of imports in the first stage. Also, suppose that, the leader foreign firm F_2 produces q_2. Then, maximizing home firm profit

$$\pi_1 = (a - q_1 - q_2)q_1,$$

we get

$$q_1 = \frac{a - q_2}{2}. \tag{12}$$

Now, maximizing foreign firm profit

$$\pi_2 = (a - q_1 - q_2 - t)q_2,$$

knowing the above quantity q_1, we get

$$q_2 = \frac{a - 2t}{2}. \tag{13}$$

Putting equation (13) into equation (12), we get

$$q_1 = \frac{a + 2t}{4}.$$

Now, we can use the above results to derive the maximum-revenue tariff. The maximum-revenue tariff is the tariff rate that maximizes the expected tariff revenue collected by the government in the home country. From the above results, the expected tariff revenue is

$$E(R) = t\frac{E(A) - 2t}{4},$$

which leads us to the following result.

Theorem 18. In the case of Stackelberg competition, with the home firm as the follower, the maximum-revenue tariff is given by

$$t^F = \frac{E(A)}{4}. \tag{14}$$

So, the output levels at equilibrium are as follows.

Theorem 19. In the case of Stackelberg competition, with the home firm as the follower, the home firm produces

$$q_1^F = \frac{2a + E(A)}{8}$$

and the foreign firm produces

$$q_2^F = \frac{2a - E(A)}{4}.$$

Thus, the aggregate quantity in the market is given by

$$Q^F = \frac{6a - E(A)}{8}$$

and the price is given by

$$p^F = \frac{2a + E(A)}{8}.$$

The following result is also obtained straightforwardly.

Theorem 20. In the case of Stackelberg competition, with the home firm as the follower, home firm's ex-ante expected profit is given by

$$E(\pi_1^F) = \left(\frac{3E(A)}{8}\right)^2 + \frac{V(A)}{16}$$

and foreign firm's ex-ante expected profit is given by

$$E(\pi_2^F) = \frac{(E(A))^2}{32} + \frac{V(A)}{8}.$$

3.4 Comparisons

In this section, we are going to compare the results obtained in each way of moving. First, we observe that, independently of the role, the sales of the home firm are increasing in the tariff, and the sales of the foreign firm are decreasing in the tariff. Furthermore, the total sales in the home country are decreasing in the tariff. Next corollary states that the domestic government imposes a lower tariff in the game where the home firm is the leader; and the tariffs are equal in the Cournot-type game and in the game where the home firm is the follower.

Corollary 21. The tariffs in the different games are related as follows:

$$t^L < t^F = t^C.$$

The total sales in the home market are higher in the game where the home firm is the leader; but, they can be lower either in the Cournot-type game or in the Stackelberg-type game where the foreign firm is the leader, depending upon the value a of the realized demand, as stated in the corollary below. This result is in contrast to the benchmark model, in which the total sales are always lower in the Cournot-type game.

Corollary 22. The total sales in the home market are related as follows:

(1) $Q^L > Q^C$ and $Q^L > Q^F$;
(2) $Q^C < Q^F$ if, and only if, $a > E(A)/2$.

In the next corollary, we compare the ex-ante expected profits of the firms in each game. We note that, in the ex-ante analysis, the worse situation for the home firm is to play a Stackelberg-type game being a follower firm; and the best situation is to play a Cournot-type game, if the variance of the demand parameter is low, and to be leader in a Stackelberg-type game, if the variance is high. This result is in contrast to the benchmark model, in which the home firm always prefers to play a Cournot-type game (see Ferreira and Ferreira (2010)). Furthermore, in the ex-ante analysis, the foreign firm prefers to be a Stackelberg leader firm, and its worse situation is to be a Stackelberg follower firm.

Corollary 23. Home firm's ex-ante expected profits are related as follows:

(1) $E\left(\pi_1^F\right) < E\left(\pi_1^L\right)$ and $E\left(\pi_1^F\right) < E\left(\pi_1^C\right)$;
(2) $E\left(\pi_1^C\right) > E\left(\pi_1^L\right)$ if, and only if, $V(A) < (E(A))^2/4$.

Foreign firm's ex-ante expected profits are related as follows: $E\left(\pi_2^L\right) < E\left(\pi_2^C\right) < E\left(\pi_2^F\right)$.

4. CONCLUSIONS

In this paper we studied an international quantity competition with demand uncertainty, where each firm is a Cournot competitor or a Stackelberg leader. We computed the maximum-revenue tariff, the quantities, the prices and the profits in each role of the model.

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